1. Compute $K_L a(\text{hr}^{-1})$. Turbine is 40 in. in diameter, rotating at 15 ft/sec peripheral speed, with an air flow of 300 SCFM.
2. Calculate $O_2$ transfer (lb/hr) under standard conditions. Saturation solubility of oxygen in the sewage liquid at 20°C is 8.45 ppm.
3. Calculate turbine horsepower corrected from Fig. 4.13.
4. Calculate blower horsepower.
5. Calculate transfer efficiency in terms of lb of $O_2$ transferred per HP × hr.

## References

1. Eckenfelder, W. W., Jr., "Water Quality Engineering for Practicing Engineers." Barnes & Noble, New York, 1970.
2. Eckenfelder, W. W., Jr., and Ford, D. L., *in* "Advances in Water Quality Improvement" (E. F. Gloyna and W. W. Eckenfelder, Jr., eds.), p. 226. Univ. of Texas Press, Austin, 1968.
3. Eckenfelder, W. W., Jr., and Ford, D. L., "Water Pollution Control." Pemberton Press, Austin and New York, 1970.
4. Eckenfelder, W. W., Jr., and O'Connor, D. J., "Biological Waste Treatment." Pergamon, Oxford, 1961.
5. Oldshue, J., "Biological Treatment of Sewage and Industrial Wastes." Van Nostrand-Reinhold, Princeton, New Jersey, 1956.
6. Quirk, T. P., unpublished report, 1962 (mentioned in Eckenfelder [1]).
7. "Standard Methods for the Examination of Water and Wastewater," 13th ed. Am. Public Health Assoc., Yearbook Publ., Chicago, Illinois, 1971.

# 5

# *Secondary Treatment: The Activated Sludge Process*

# 1. Introduction

The heading secondary treatment encompasses all biological treatment processes of wastewaters, both aerobic and anaerobic. In this chapter the activated sludge process is studied in detail, and the mathematical models developed are applicable, with minor changes, to all aerobic processes described in Chapter 6.

The activated sludge process has been utilized for treatment of both domestic and industrial wastewaters for approximately half a century. Design of activated sludge plants was carried out to a large extent in an empirical manner. It was only after the early 1960's that a more rational approach to the design of activated sludge systems was developed. This process originated from the observation made a long time ago that whenever wastewater, either domestic or industrial, is aerated for a period of time, the content of organic matter is reduced, and at the same time a flocculent sludge is formed.

Microscopic examination of this sludge reveals that it is formed by a heterogeneous population of microorganisms, which changes continually in nature in response to variation in the composition of the wastewater and environmental conditions. Microorganisms present are unicellular bacteria, fungi, algae, protozoa, and rotifers. Of these, bacteria are possibly the most important, being found in all types of biological treatment processes.

The purpose of this chapter is to discuss the design principles for the activated sludge process and to apply them to design of treatment plants. This involves development of fundamental design information from laboratory scale reactors. The approach utilized is based mainly on the work of Eckenfelder and associates.

The activated sludge process has been developed as a continuous operation by recycling the biological sludge. A flow diagram of this continuous process



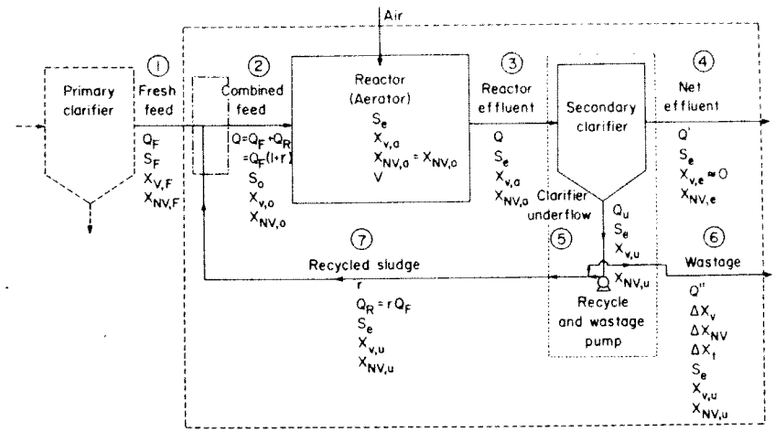


***Fig. 5.1. Conventional activated sludge process. (See Table 5.1 for a definition of symbols.)***

is shown in Fig. 5.1. All important process variables are indicated in Fig. 5.1 and defined in Table 5.1. These should be carefully examined by the reader.

In Fig. 5.1, compositions of the different streams (numbered 1–7) are characterized by three types of concentrations:

1. *Concentration of soluble BOD.* Denoted by the symbol $S_i$, where subscript $i$ refers to the specific stream under consideration, as indicated in Table 5.1. Soluble BOD comprises mainly carbonaceous materials in solution.

2. *Concentrations of volatile suspended solids (VSS).* These are denoted by symbol $X_{v,i}$, where subscript $v$ stands for volatile, and subscript $i$ refers to the specific stream in question (Table 5.1). VSS correspond to the biological sludge, constituted by the heterogeneous population of microorganisms. Experimental determination of VSS is performed by measuring the loss of weight of total suspended solids (TSS), after incineration in a laboratory oven at 600°C. This loss of weight corresponds mainly to volatilization of biological sludge. Remaining solids after incineration at 600°C are referred to as nonvolatile suspended solids. Their nature is distinct from those in the biological sludge, being constituted of nonliving matter of both inorganic and organic nature.

3. *Concentrations of nonvolatile suspended solids (NVSS).* These are denoted by symbol $X_{NV,i}$, where $NV$ stands for nonvolatile, and $i$ refers to the specific stream in question.

Therefore

$$\text{TSS} = \text{VSS} + \text{NVSS}$$

Total suspended solids = Volatile suspended solids + Nonvolatile suspended solids

**TABLE 5.1**
**Definition of Symbols Used in Fig. 5.1**

Key

For suspended solids double subscripts are utilized, e.g., $X_{v,i}$, $X_{NV,i}$.

The first subscript ($v$ or $NV$) designates volatile and nonvolatile suspended solids, respectively. The second subscript ($i$) refers to the specific stream in question:

$F$, fresh feed [stream 1]
$o$, combined feed [stream 2]
$a$, reactor effluent [stream 3]
$e$, net effluent [stream 4]
$u$, underflow from secondary clarifier [stream 5]

Symbols

1. Flow rates
   $Q_F$, fresh feed; MGD (million gallons per day) [stream 1]
   $Q_R$, recycle; MGD [stream 7]
   $r$, recycle ratio; dimensionless ($r = Q_R/Q_F$)
   $Q$, combined feed; MGD; $Q = Q_F + Q_R = Q_F(1+r)$ [stream 2]
   (MGD of combined feed = MGD of reactor effluent, i.e., $Q$ [stream 2] = $Q$ [stream 3])
   $Q'$, net effluent; MGD [stream 4]
   $Q''$, wastage; MGD [stream 6] (Notice that $Q_F = Q' + Q''$)
   $Q_u$, clarifier underflow; MGD; $Q_u = Q'' + Q_R = Q'' + rQ_F$ [stream 5]

2. Concentrations (mg/liter) of soluble BOD
   $S_F$, soluble BOD of fresh feed
   $S_o$, soluble BOD of combined feed
   $S_e$, soluble BOD of effluent

3. Concentrations (mg/liter) of volatile suspended solids (VSS)
   $X_{V,F}$, VSS in fresh feed
   $X_{v,o}$, VSS in combined feed
   $X_{v,a}$, VSS in reactor. This also is equal to concentration of VSS in reactor effluent
   $X_{v,u}$, VSS in secondary clarifier underflow
   $X_{v,e}$, VSS in net effluent (take $X_{v,e} \approx 0$)

4. Concentrations (mg/liter) of nonvolatile suspended solids (NVSS)
   $X_{NV,F}$, NVSS in fresh feed
   $X_{NV,o}$, NVSS in combined feed
   $X_{NV,a}$, NVSS in reactor ($X_{NV,a} = X_{NV,o}$). This also equals concentration of NVSS in reactor effluent
   $X_{NV,u}$, NVSS in secondary clarifier underflow
   $X_{NV,e}$, NVSS in net effluent

5. Wastage (lb/day)
   $\Delta X_v$, net yield of MLVSS in reactor (wastage of MLVSS)
   $\Delta X_{NV}$, wastage of NVSS
   $\Delta X_t$, total sludge yield: $\Delta X_t = \Delta X_v + \Delta X_{NV} + Q_F X_{V,F}$

6. Reactor volume
   $V$, reactor volume, MG (million gallons)



A description of the flowsheet in Fig. 5.1 follows, with emphasis on concentrations of (1) soluble BOD, (2) volatile suspended solids, and (3) nonvolatile suspended solids for the different streams.

1. *Soluble BOD.* Fresh feed, i.e., the wastewater to be treated [stream 1], enters the process with a value of soluble BOD denoted as $S_F$. Purpose of the treatment is to reduce this value to $S_e$ (effluent BOD in stream 4) by oxidation through aerobic biological degradation of organic matter in the wastewater.

In the conventional activated sludge process, a reduction of soluble BOD to 5–10% of its value in the fresh feed is usually accomplished, i.e., $S_e$ = 5–10% of $S_F$. This means a soluble BOD removal efficiency of 90–95%.

Fresh feed is combined with recycled sludge [stream 7] and enters the aerator (combined feed, stream 2). Biological sludge is continuously formed in the aerator. It is usually desirable to operate the reactor at steady state and under complete mixing conditions. These two assumptions are made in most mathematical models hence. Concentration of soluble BOD in the reactor liquor is denoted as $S_e$. Under steady state and complete mixing conditions the concentration of soluble BOD in reactor effluent [stream 3] also equals $S_e$.

Reactor effluent enters the secondary clarifier as indicated in Fig. 5.1. Concentration of soluble BOD is the same in clarifier underflow [stream 5] and net effluent [stream 4], i.e., $S_e$. Clarifier underflow is split into two streams: wastage [stream 6] and recycled sludge [stream 7]. For both these streams, the concentration of soluble BOD has the same value, $S_e$. The recycled sludge stream is then combined with fresh feed to form the combined feed. Concentration of soluble BOD in combined feed, designated as $S_o$, is calculated by a material balance at the junction point of streams 1, 2, and 7. This balance is written in Section 7.3.

2. *Volatile suspended solids (VSS).* At steady state, concentration of biological sludge in the reactor is kept constant at all times. In the conventional activated sludge process this concentration, designated as $X_{v,a}$, where the second subscript $a$ refers to the aerator, is usually selected between 2000 and 3000 mg/liter. Since complete mixing conditions are postulated to exist in the reactor, volatile suspended solids in it are referred to as MLVSS (mixed liquor volatile suspended solids). Similarly, nonvolatile suspended solids in the reactor, being also completely mixed, are referred to as MLNVSS (mixed liquor nonvolatile suspended solids). Total suspended solids in the reactor are designated as MLTSS (mixed liquor total suspended solids).
Therefore

$$\text{MLTSS} = \text{MLVSS} + \text{MLNVSS}$$

Mixed liquor total suspended solids = mixed liquor volatile suspended solids

+ mixed liquor nonvolatile suspended solids

Concentration of VSS in fresh feed ($X_{V,F}$) is negligible in many cases, since no appreciable amount of aeration has taken place at this stage. VSS is produced continuously in the aerator, owing to synthesis of biological matter, and withdrawn continuously with reactor effluent.

In order to maintain a constant concentration of MLVSS in the reactor, most of the clarifier underflow is recycled back. Recycle ratio $r$ is calculated by material balance (Section 7.2) in order to maintain a constant selected concentration $X_{v,a}$ of MLVSS within the reactor at all times. Owing to synthesis of biological matter there is a *net* yield of MLVSS in the reactor ($\Delta X_v$, lb/day). Therefore to maintain constant concentration of MLVSS in the reactor at all times, it is necessary to remove from the system a mass of MLVSS (lb/day) equal to this net yield $\Delta X_v$. This is done by wastage of sludge [stream 6]. Although continuous wastage is indicated in Fig. 5.1, in practice it is usually an intermittent operation. It is simpler to write material balances for a steady state operation; thus continuous wastage is assumed in the remainder of this chapter. Intermittent wastage implies the assumption of unsteady state operation. Since the wastage stream is usually small by comparison with the recycle, assumption of continuous wastage does not introduce, in general, an appreciable error in the overall material balance. Concentration of VSS in the reactor effluent [stream 3] is also $X_{v,a}$, since complete mixing and steady state conditions are assumed.

Reactor effluent flows into the secondary clarifier. Underflow from the latter [stream 5] is a slurry containing a concentration of VSS designated as $X_{v,u}$ ($X_{v,u} > X_{v,a}$). The value of $X_{v,u}$ is selected by the designer, with clarifier being sized to yield this specified value. Usually $X_{v,u}$ is selected between 10,000 and 15,000 mg/liter of MLVSS. Concentrations of VSS in wastage and recycled sludge are also equal to $X_{v,u}$. In the net effluent from the secondary clarifier, concentration of VSS ($X_{v,e}$) is neglected in development of material balances in this chapter. This implies that complete separation of VSS is assumed to take place in the secondary clarifier (i.e., $X_{v,e} \approx 0$). This is usually a good assumption. Concentration of VSS in combined feed, $X_{v,o}$, is calculated by a material balance at the junction point of streams 1, 2, and 7. This balance is written in Section 4.5.

3. *Nonvolatile suspended solids* (*NVSS*). Concentration of MLNVSS in the aerator is denoted as $X_{NV,a}$ and is equal to those in both combined feed and reactor effluent. This is so because complete mixing is assumed and there is no production of NVSS in the aerator (unlike the net yield of VSS). Thus

$$X_{NV,a} = X_{NV,o}$$

Concentration of NVSS in fresh feed is designated as $X_{NV,F}$ and that in the recycled sludge as $X_{NV,u}$ (same as in underflow from secondary clarifier). In



the combined feed this concentration is denoted as $X_{NV,o}$ and is calculated by a material balance which is written in Section 4.5.

Some of the NVSS in the reactor effluent is also separated by sedimentation in the secondary clarifier. Concentration of NVSS in clarifier underflow is denoted as $X_{NV,u}$ and that in net effluent as $X_{NV,e}$.

In wasted sludge, besides the $\Delta X_v$ lb/day of VSS there is also some nonvolatile sludge ($\Delta X_{NV}$ lb/day) resulting from partial sedimentation of NVSS in the secondary clarifier. In addition, there is the biological sludge introduced continuously with the fresh feed ($Q_F X_{V,F}$). Frequently, term $Q_F X_{V,F}$ is negligible since $X_{V,F}$ is usually very small. Total sludge wasted, $\Delta X_t$ lb/day, is [Eq. (5.1)]

$$\Delta X_t = \Delta X_v + \Delta X_{NV} + Q_F X_{V,F} \tag{5.1}$$

Respective concentrations of VSS, NVSS, and soluble BOD are the same for clarifier underflow, wastage stream, and recycle sludge, being denoted, respectively, as $X_{v,u}$, $X_{NV,u}$, and $S_e$.

In summary, concentrations of VSS, NVSS, and soluble BOD in combined feed ($X_{v,o}$, $X_{NV,o}$, and $S_o$, respectively) are obtained by material balances around the junction point of fresh feed and recycled sludge streams. These material balances are written in Sections 4.5 and 7.3.

From an overall balance for the wastewater [Eq. (5.2)],

$$Q_F = Q' + Q'' \tag{5.2}$$

Wastewater flows are usually expressed in millions of gallons per day (MGD).

Recycle ratio $r$ is defined as

$$r = Q_R/Q_F = \text{recycle wastewater, MGD/fresh wastewater, MGD} \tag{5.3}$$

$$\therefore Q_R = rQ_F \tag{5.4}$$

Since combined feed $Q$ is equal to fresh feed *plus* recycle,

$$Q = Q_F + Q_R = Q_F(1+r) \tag{5.5}$$

Hence, the density of all liquor streams in Fig. 5.1 is assumed equal to that of water at ambient temperature (8.34 lb/gal).* This is a good approximation since relatively dilute aqueous solutions are involved.

## 2. Mathematical Modeling of Activated Sludge Process

It is desirable to portray this process by a mathematical model and then to determine parameters utilized in mathematical equations from experimental data obtained utilizing a series of bench scale laboratory reactors. Relationships which are pertinent to the development of this mathematical model fall

* This value is approximately 10.0 lb/gal when imperial gallons are utilized.

into three groups: (1) kinetics relationships; (2) material balance relationships—material balance for determination of oxygen utilization and of net yield of MLVSS; and (3) relationship for optimum settling conditions of sludge.

# 3. Kinetics Relationships

## 3.1. INTRODUCTION

Study of kinetics of aerobic biological treatment yields the rate at which microorganisms degrade a specific waste, and therefore provides the basic information required for sizing biological aerobic reactors. This study is conveniently performed in a laboratory scale batch reactor. Figure 5.2 shows

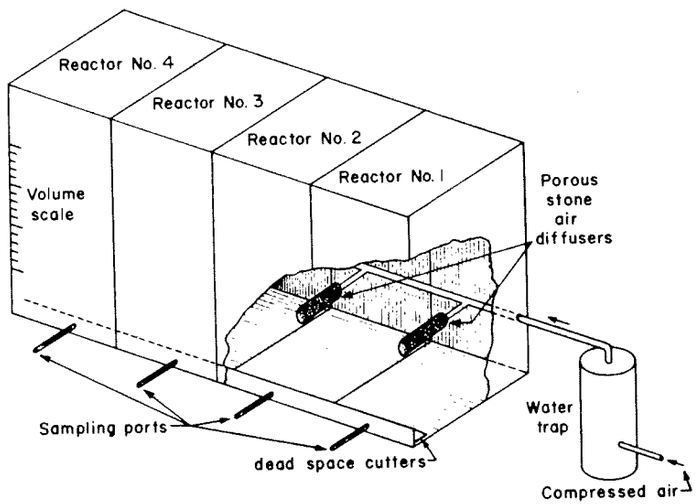


*Fig. 5.2. Batch reactor.*

a diagram of four units operating in parallel, each with a capacity of approximately 2.0 liters [3]. Reactors are built of plexiglass. Wastewater containing a seed of microorganisms* is introduced into the reactors, and compressed air is blown into the system. The biological sludge (MLVSS) is kept in a state of complete mixing due to agitation provided by air blown into the system.

* Seed is either a mass of biological sludge taken from an operating activated sludge plant, or settled sewage.



BOD of wastewater (or COD, TOD, TOC) is determined at selected time intervals by withdrawing samples for the analysis. The mass of accumulated biological sludge (MLVSS) is also determined at these same time intervals by measuring the concentration of MLVSS in withdrawn samples and reading the volume of liquor in the reactor as indicated by the volume scale. Typical curves for decrease of BOD and variation of the amount of MLVSS with time are presented in Fig. 5.3.

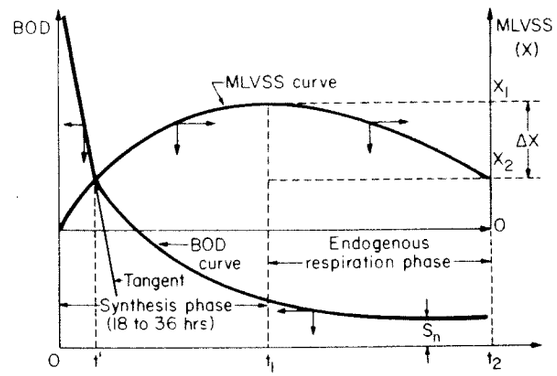


*Fig. 5.3. Typical BOD and MLVSS curves for a batch reactor.*

BOD of the wastewater, which is a measure of organic biodegradable matter concentration, decreases with time as the organic matter is oxidized. A plateau is eventually reached corresponding to the amount of nonbiodegradable matter ($S_n$).

Concentration of MLVSS increases at first (from time 0 to time $t_1$) during the period when a substantial concentration of substrate (relativley high BOD) is present to provide abundant food to sustain growth of microorganisms. This growth corresponds to the synthesis of new microorganism cells, indicated in Fig. 5.3 as "synthesis phase." After time $t_1$ when substrate concentration is considerably depleted, there is not enough food left to sustain growth of microorganisms. At this time, microorganisms start consuming their "fellow microorganisms" as food. As this "cannibalistic feast" proceeds, concentration of MLVSS drops when the rate of destruction of microorganism cells exceeds that of synthesis of new cells. This corresponds to the "endogenous respiration phase." The maximum on the MLVSS curve corresponds to time $t_1$, when these two rates are exactly equal. Distance $\Delta X$ corresponds to the *net* reduction of MLVSS concentration from $t_1$ to $t_2$.

There are two fundamental differences between operation of continuous (Fig. 5.1) and batch reactors (Fig. 5.2): (1) Contrary to what happens in the

batch reactor, BOD of the wastewater in the continuous reactor operating at steady state conditions remains constant ($S_e$). This corresponds generally to a low substrate concentration, since the biological reactor is usually designed for removing most of the influent BOD. (2) Contrary to what happens in the batch reactor, concentration of MLVSS in the continuous reactor operating at steady state is kept constant ($X_{v,a}$) at a selected value. Maintenance of this constant $X_{v,a}$ is obtained by providing the calculated amount of concentrated return sludge. The material balance for MLVSS, necessary to arrive at required recycle ratio for this purpose, is presented in Section 7.2.

Kinetic data obtained from the batch reactor is portrayed by the Michaelis–Menten relationship, which is studied in Section 8. Two important corollaries of this relationship are postulated next, the second one being utilized for design of the continuous biological reactor.

1. At high substrate concentrations, BOD removal follows zero-order kinetics. This means that the rate of removal is essentially constant, independent of substrate concentration. This situation is found in early stages of the batch reactor operation, when substrate concentration is still very high (high BOD). This corresponds to the section of the BOD curve (Fig. 5.3) from time zero to approximately time $t'$. In this region, the tangent to the BOD curve, which equals the rate of substrate removal, coincides essentially with the curve itself (constant slope).

2. BOD removal at low substrate concentrations (corresponding to BOD values below 500 mg/liter) follows first-order kinetics. This means that rate of removal is proportional to remaining substrate concentration. This corresponds to the section of the BOD curve beyond time $t'$. Slope of the BOD curve (which equals rate of substrate removal) decreases with time as the BOD value is lowered. A plot of these slopes vs. corresponding BOD values yields a straight line relationship, which is discussed in Section 3.2. Thus in this region, rate of substrate removal is directly proportional to its concentration (first-order kinetics).

## 3.2. FORMULATION OF THE CONTINUOUS REACTOR

Since for the continuous reactor operating substrate concentrations ($S_e$) are considerably below 500 mg/liter ($BOD_5$), first-order kinetics is assumed in the formulation. Consider the continuous reactor operating under steady state and complete mixing conditions. This situation is illustrated by Fig. 5.4.

Assuming that rate of substrate removal $dS/dt$ follows first-order kinetics,*

$$dS/dt = -KS \tag{5.6}$$

It is customary to express substrate removal rate per mg/liter of MLVSS

* Minus sign in Eq. (5.6) is required since $dS/dt < 0$, whereas $S > 0$.



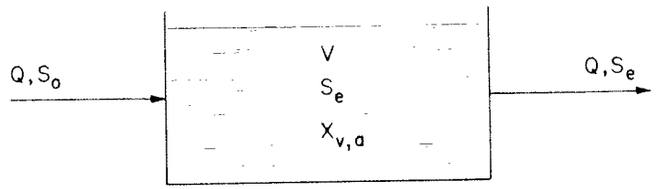


*Fig. 5.4. Simplified diagram for continuous reactor.*

present in the reactor. Let $X_{v,a}$ be this MLVSS concentration. Equation (5.6) is then rewritten

$$(1/X_{v,a})(dS/dt) = -kS \tag{5.7}$$

The relationship between $K$ and $k$ is

$$K = kX_{v,a} \tag{5.8}$$

From Eq. (5.7)

$$dS/dt = -kX_{v,a}S \tag{5.9}$$

$k$ is the substrate removal rate constant. For time $t$ equal to residence time in the continuous reactor, concentration $S$ corresponds to $S_e$, and Eq. (5.9) becomes

$$(dS/dt)_{\text{cont. reactor}} = -kX_{v,a}S_e \tag{5.10}$$

The following material balance for substrate is written for the reactor in Fig. 5.4.

$$\begin{aligned}\text{Change of substrate in reactor} = {} & \text{increase due to influent flow} \\ & - \text{decrease due to effluent flow} \\ & - \text{decrease due to reaction}\end{aligned} \tag{5.11}$$

Under steady state conditions,

$$\text{Change of substrate in reactor} = 0 \tag{5.12}$$

$$\text{Increase due to influent flow} = QS_o \tag{5.13}$$

and

$$\text{Decrease due to effluent flow} = QS_e \tag{5.14}$$

According to Eq. (5.10), the decrease in the amount of substrate due to the reaction is $kX_{v,a}S_e$ [minus sign already included in Eq. (5.11)]. Before substituting in Eq. (5.11) this value is multiplied by reactor volume $V$, since $kX_{v,a}S_e$ represents decrease per unit volume.

$$\text{Decrease due to reaction} = kX_{v,a}S_eV \tag{5.15}$$

Substitution of values given by Eqs. (5.12)–(5.15) in Eq. (5.11) yields after manipulation

$$(Q/V)[(S_o - S_e)/X_{v,a}] = kS_e \tag{5.16}$$

However,

$$t = V/Q = \frac{\text{Mgal}}{(\text{Mgal/day})} = \text{day} = \text{residence time } (t) \text{ in the reactor} \tag{5.17}$$

Consequently, Eq. (5.16) is

$$(S_o - S_e)/X_{v,a}t = kS_e \tag{5.18}$$

Term $(S_o - S_e)/X_{v,a}t$ which also appears in other formulations is the substrate removal rate. It corresponds to rate of removal of substrate in the continuous reactor per mg/liter of MLVSS present. Units are

$$(S_o - S_e)/X_{v,a}t = \frac{\text{mg/liter of BOD removed}}{(\text{mg/liter of MLVSS})(\text{day})}$$

$$= \text{mg BOD removed/(day)(mg MLVSS)}$$

$$= \text{lb BOD removed/(day)(lb MLVSS)}$$

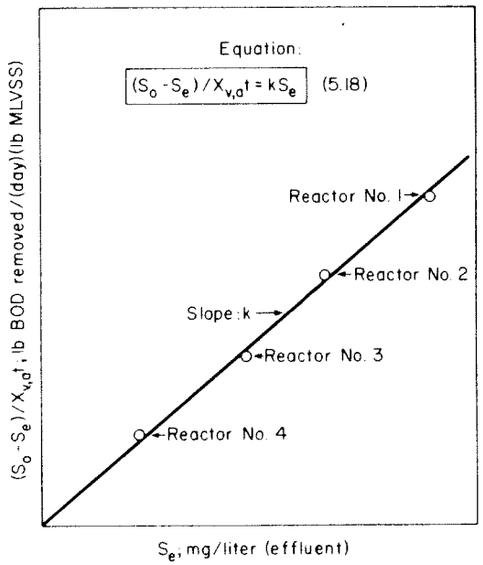


*Fig. 5.5. Graphical determination of k (four continuous lab scale reactors).*



Equation (5.18) indicates that the substrate removal rate is proportional to substrate concentration $S_e$ (first-order kinetics). Substrate removal rate constant $k$ ($\text{day}^{-1}$) is determined according to Eq. (5.18) from a plot of $(S_o - S_e)/X_{v,a}t$ vs. $S_e$. Figure 5.5 shows a graph of data obtained from four continuous laboratory reactors operating at steady state conditions. A numerical application is presented in Section 6.4 (Example 5.5).

Data plotted in Fig. 5.5 yield a straight line passing through the origin, assuming applicability of the mathematical model in Eq. (5.18). The left-hand member, $(S_o - S_e)/X_{v,a}t$, vanishes as $t$ approaches infinity (infinite residence time). Consequently, term $S_e$ in the right-hand member approaches zero since $k \neq 0$. This corresponds to *complete* removal of substrate, which is not always the case since some substrates cannot be completely degraded by the aerobic biological process, even at infinite residence time. In these cases, the straight line cuts the abscissa at a value of $S_e > 0$ corresponding to the concentration of nonbiodegradable matter. An example of this situation is shown in Fig. 5.14 (Section 6.4, Example 5.5).

When nonbiodegradable matter is present, Eq. (5.18) is modified to Eq. (5.19).

$$(S_o - S_e)/X_{v,a}t = k(S_e - S_n) \tag{5.19}$$

where $S_n$ is the concentration of nonbiodegradable matter.

# 4. Material Balance Relationships

## 4.1. DESIGN PARAMETERS CORRESPONDING TO NET YIELD OF MLVSS AND OXYGEN REQUIREMENTS FOR AEROBIC BIOLOGICAL DEGRADATION OF WASTES

### 4.1.1. Introduction: Mechanism of Aerobic Biological Degradation

Accumulation of MLVSS and utilization of oxygen are two important elements needed for design of aerobic biological reactors. To obtain mathematical models which yield these two values, several design parameters designated by symbols $a'$, $\bar{a}$, $a$, $b$, and $b'$ are defined in this section. The approach followed is that proposed by Eckenfelder and associates [1–3].

Evaluation of these parameters is accomplished by using bench scale continuous biological reactors (Section 6). In the discussion which follows, numerical values for these parameters are utilized for clarification of some concepts. These values are obtained by techniques discussed in Section 6.

To arrive at the definition of these parameters, the basic mechanism of aerobic degradation of a substrate must be understood. Consider that a substrate is charged to a batch reactor (Fig. 5.2), and that curves for BOD

removal and MLVSS concentration are obtained (Fig. 5.3). For clarification, take the hypothetical case of pure lactose as substrate. Assume that a lactose solution is charged to the batch reactor with a seed of microorganism, and that compressed air is bubbled through the solution. Let initial concentration of lactose be equal to 1050 mg/liter. Suppose that after a time $t$ this concentration is reduced to 50 mg/liter. Thus substrate removed is $1050 - 50 = 1000$ mg/liter. Assume that ThOD is utilized as a measure of lactose concentration.* The chemical equation corresponding to ThOD for lactose is [Eq. (5.20)]†

$$(CH_2O) + O_2 \rightarrow CO_2 + H_2O \qquad (5.20)$$

Molecular weight: 30 32

Thus, the initial ThOD of the solution is $(32/30) \times 1050 = 1120$ mg/liter. After time $t$, remaining ThOD is $(32/30) \times 50 = 53.3$ mg/liter. Therefore, ThOD removed is

$$1120 - 53.3 = 1066.7 \text{ mg/liter}$$

or

$$(32/30)(1050 - 50) = 1066.7 \text{ mg/liter} \qquad (5.21)$$

Thus, ThOD and substrate removed are proportional, the proportionality constant being $32/30 = 1.07$. Since ThOD is correlated to COD, BOD, etc., one may also express substrate removal in terms of these parameters.

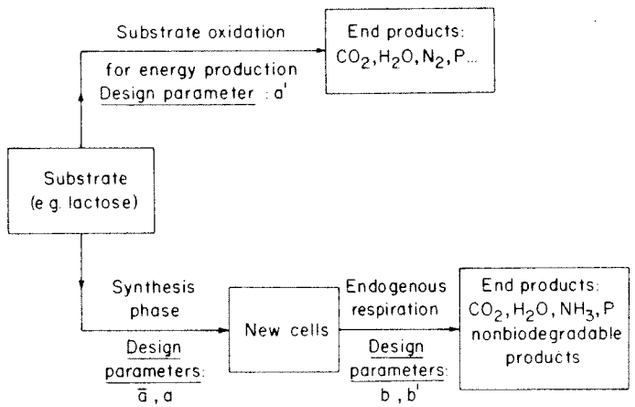


***Fig. 5.6. Mechanism of aerobic biological degradation.***

* As discussed in Chapter 2, ThOD is only utilized in rare cases when complete analysis of the wastewater is known.

† For simplicity in Eq. (5.20), lactose is represented by one sugar unit $(CH_2O)$. Multiplying this unit by a factor of 12 one obtains

$$12(CH_2O) = C_{12}H_{22}O_{11} \cdot H_2O$$

which is the molecular formula for lactose.



Mechanism of aerobic biological degradation of a substrate is represented diagrammatically by Fig. 5.6. Design parameters ($a'$, $\bar{a}$, $a$, $b$, and $b'$) indicated in Fig. 5.6 are defined in Sections 4.1.2–4.1.9. These values are calculated from laboratory data (Section 6).

Figure 5.6 indicates that substrate is removed during the biological process in two ways.

1. Part of the substrate, after being consumed as food by microorganisms, is utilized to synthesize new microorganism cells. This corresponds to the synthesis phase. For the lactose example, this synthesis corresponds to‡

$$5(CH_2O) \xrightarrow{\text{synthesis}} C_5H_7NO_2 \qquad (5.22)$$

Molecular weight: $5 \times 30 = 150$ 113 (MLVSS)

Intermediate steps in Eq. (5.22) are complicated and irrelevant. The empirical formula $C_5H_7NO_2$ corresponds to the average composition of MLVSS cells. Nitrogen is needed for synthesis and must be provided. From the approximate empirical formula $C_5H_7NO_2$ it follows that % of nitrogen in the MLVSS cells is $(14/113) \times 100 = 12.4\%$.

2. The remainder of the substrate is oxidized, terminal products being $CO_2$ and $H_2O$. In the lactose example, this substrate oxidation corresponds to Eq. (5.20). This terminal oxidation process is extremely important in the production of cellular energy utilized by the cells to maintain their normal functions, such as synthesis, reproduction, and mobility. Assume that 65% of the lactose removed (i.e., 65% of 1000 mg/liter = 650 mg/liter) is dioxized to provide energy requirements, and that 35% (i.e., 350 mg/liter) is utilized in the synthesis of new cell matter. Since there is a proportionality constant relating substrate and ThOD removals [factor (32/30) in Eq. (5.20) for lactose], it follows that 65% of the ThOD removed is utilized for energy generation and 35% for synthesis of new cells. Similar statements are valid in terms of COD and other parameters defined in Chapter 2 (Sections 2 and 3).

‡ Phosphorus is also utilized in the synthesis and becomes a constituent of cell matter. The % of phosphorus in the MLVSS cells is approximately 2%, so a more accurate empirical formula for the MLVSS cells is $C_5H_7NO_2P_n$ where $n$ is given by (atomic weight of phosphorus = 31)

$$31n/(113 + 31n) = 2/100$$

$$\therefore n = 0.074$$

or $C_5H_7NO_2P_{0.074}$. Nitrogen and phosphorus needed are provided by addition of ammonium phosphate to the wastewater, if it does not already contain the nitrogen and phosphorus required.

### 4.1.2. Definition of Parameter $\bar{a}$ (Synthesis Phase)

Let $\bar{a}$ be the fraction of substrate removed that is utilized for synthesis (namely, $\bar{a} = 0.35$ in lactose example). Due to the proportionality between removal of substrate and those of ThOD, COD, or BOD, $\bar{a}$ also represents fractions of ThOD (or COD, BOD) utilized for synthesis of new cells. Therefore,

$$\begin{aligned}\bar{a} &= \text{lb of substrate removed utilized for synthesis/lb of total substrate removed}\\ &= \text{lb ThOD removed for synthesis/lb of total ThOD removed}\\ &= \text{lb COD removed for synthesis/lb total COD removed}\\ &= \text{lb BOD removed for synthesis/lb total BOD removed}\end{aligned} \tag{5.23}$$

The numerical value of $\bar{a}$ is independent of parameters utilized for expressing substrate removal, since $\bar{a}$ represents the fraction of substrate removed utilized for synthesis, and is therefore a dimensionless quantity. The same conversion factor for changing parameters in which substrate removal is to be expressed appears simultaneously in the numerator and denominator of Eq. (5.23), and therefore cancels out.

Parameter $\bar{a}$ *does not* appear in the final formulation of aerobic processes developed in Section 6. Instead parameter $a$, which is related to $\bar{a}$, is utilized.

### 4.1.3. Definition of Parameter $a'$ (Oxidation)

Let $a'$ be the fraction of substrate removed utilized for energy production (namely, $a' = 0.65$ in lactose example).
Therefore,

$$\bar{a} + a' = 1.0 \tag{5.24}$$

where

$$\begin{aligned}a' &= \text{lb of substrate removed utilized for energy/lb of total substrate removed}\\ &= \text{lb ThOD removed for energy/lb total ThOD removed}\\ &= \text{lb COD removed for energy/lb total COD removed}\\ &= \text{lb BOD removed for energy/lb total BOD removed}\end{aligned} \tag{5.25}$$

The numerical value of $a'$ defined by Eq. (5.25) is independent of the parameters utilized for expressing substrate removal. The same observations made for $\bar{a}$ are applicable here.

SUMMARY For the lactose example

Total substrate removed: 1000 mg/liter
Total ThOD removed: $32/30 \times 1000 = 1066.7$ mg/liter. These removals take place in two ways:



(1) Synthesis:

$$5(CH_2O) \rightarrow C_5H_7NO_2$$

Substrate removed utilized for synthesis:

$$(0.35)(1000) = 350 \text{ mg/liter}$$

ThOD removed for synthesis:

$$(0.35)(1066.7) = 373.3 \text{ mg/liter}$$

[Ratios, $350/1000 = 373.3/1066.7 = 0.35 = \bar{a}$]

(2) Energy production:

$$(CH_2O) + O_2 \rightarrow CO_2 + H_2O$$

Substrate removed utilized for energy production:

$$(0.65)(1000) = 650 \text{ mg/liter}$$

ThOD removed for energy production:

$$(0.65)(1066.7) = 693.4 \text{ mg/liter}$$

[Ratios, $650/1000 = 693.4/1066.7 = 0.65 = a'$]

From Eq. (5.20) ThOD removed for energy production equals the lb of oxygen utilized for oxidation of substrate. Therefore the definition of $a'$ (in terms of ThOD) given by Eq. (5.25) is reformulated as

$$a' = a'_{\text{ThOD}} = \text{lb of } O_2 \text{ utilized in oxidation of substrate/lb of total ThOD removed} \tag{5.26}$$

i.e., $a'$ is equal to the lb of oxygen utilized in energy production per lb of total ThOD removed.

Therefore from Eq. (5.26),

$$\begin{aligned}\text{lb } O_2 \text{ (for energy)} &= a'(\text{lb total ThOD removed})\\ &= a'_{\text{ThOD}}(\text{lb total ThOD removed})\end{aligned} \tag{5.27}$$

Writing the right-hand member of Eq. (5.27) in terms of COD, BOD, and TOC by utilizing ratios ThOD/COD, ThOD/BOD, etc., yields

$$\begin{aligned}\text{lb } O_2 \text{ (for energy)} &= a'(\text{lb total COD removed})(\text{ThOD/COD})\\ &= a'(\text{lb total BOD removed})(\text{ThOD/BOD})\end{aligned} \tag{5.28}$$

Define subscript values of $a'$ as

$$a'_{\text{COD}} = a'(\text{ThOD/COD}) \tag{5.29}$$

$$a'_{\text{BOD}} = a'(\text{ThOD/BOD}) \tag{5.30}$$

(where $a'$ without the subscript stands for value $a' = a'_{\text{ThOD}}$).

Combining Eqs. (5.27) and (5.28) with Eqs. (5.29) and (5.30),

$$\begin{aligned} \text{lb } O_2 \text{ (for energy)} &= a'_{\text{ThOD}}(\text{lb total ThOD removed}) \\ &= a'_{\text{COD}}(\text{lb total COD removed}) \\ &= a'_{\text{BOD}}(\text{lb total BOD removed}) \end{aligned} \tag{5.31}$$

Hence, whenever parameter $a'$ is utilized for calculation of oxygen requirements, no subscripts are indicated. An appropriate value of $a'$ is chosen to be compatible with parameters for expressing substrate removal. From Eq. (5.31) it follows that $a'$ equals the lb of oxygen utilized for energy production per lb of substrate removed (removal in terms of ThOD, COD, and TOD).

Utilization of subscripts COD and BOD for $a'$ may seem inconsistent since $a'$ is thought of as a ratio, and therefore its numerical value should be *independent* of parameters utilized for expressing removal. However, this independence applies only to values of $a'$ as defined by Eq. (5.25). In Eq. (5.25) the same conversion factor for parameters expressing removal appears simultaneously in the numerator and denominator, and therefore cancels out. From Eq. (5.31), however, it follows that a modified definition of $a'$ is being utilized, i.e.,

$$a'_{\text{ThOD}} = a' = \text{lb } O_2 \text{ (for energy)/lb total ThOD removed} \tag{5.32}$$

$$a'_{\text{COD}} = \text{lb } O_2 \text{ (for energy)/lb total COD removed} \tag{5.33}$$

$$a'_{\text{BOD}} = \text{lb } O_2 \text{ (for energy)/lb total BOD removed} \tag{5.34}$$

The numerical value of the numerators in Eqs. (5.32), (5.33), and (5.34) is the same (lb of oxygen utilized for energy requirements). Values of denominators, however, vary depending on choice of parameters for expressing substrate removal. Consequently, numerical values of $a'$ from Eqs. (5.32), (5.33), and (5.34) are different from each other. Therefore, utilization of subscripts is justified.

Furthermore, *only* the value of $a'$ given by Eq. (5.32) is numerically equal to the ratios defined by Eq. (5.25), i.e., $a'_{\text{ThOD}} = a'$. Values of $a'$ given by Eqs. (5.33) and (5.34) are not only different from each other, but also neither equals the fraction of substrate removed utilized in energy production.

### 4.1.4. Definition of Parameter *a* (Synthesis Phase)

Parameter $a$, related to $\bar{a}$, is defined as

$$a = \text{lb of MLVSS produced/lb of total substrate removed} \tag{5.35}$$

Consequently, $a$ represents yield of biological sludge per lb of total substrate removed.



The relationship between parameters $\bar{a}$ and $a$ is arrived at by consideration of the lactose example [Eq. (5.22)]. It is assumed that 350 mg/liter (35% of the total 1000 mg/liter of lactose removed) are utilized for the synthesis indicated by Eq. (5.22). Yield of MLVSS is calculated as

$$\begin{aligned} &\text{MLVSS produced per 1000 mg of total substrate removed} \\ &= [(0.35)(1000)](113/150) \\ &= 263.7 \text{ mg/liter} \end{aligned} \tag{5.36}$$

Therefore, from Eq. (5.36) one obtains

$$\begin{aligned} a &= \text{lb MLVSS produced/lb of total substrate removed} \\ &= [(0.35)(1000)](113/150)/1000 = 263.7/1000 = 0.2637 \end{aligned} \tag{5.37}$$

i.e., 263.7 mg/liter of MLVSS are produced per 1000 mg/liter of lactose removed; thus $a = 263.7/1000 = 0.2637$.

The relationship between $a$ and $\bar{a}$ from Eq. (5.37) for the lactose example is

$$a = \bar{a}(113/150)$$

$$\therefore \bar{a} = a(150/113)$$

where 113/150 is the stoichiometric ratio for Eq. (5.22). Substitution of this value of $\bar{a}$ in Eq. (5.24) yields

$$(150/113)a + a' = 1.0 \tag{5.38}$$

Parameter $a$ may be written in terms of total ThOD removed. Let $a_{\text{ThOD}}$ be the numerical value of $a$ expressed in this manner.

$$a_{\text{ThOD}} = \text{lb MLVSS produced/lb of total ThOD removed} \tag{5.39}$$

Ratio $a/a_{\text{ThOD}}$ from Eqs. (5.35) and (5.39), taking into account the stoichiometric ratio 32/30 in Eq. (5.20), is

$$\begin{aligned} a/a_{\text{ThOD}} &= \text{Eq. (5.35)/Eq. (5.39)} \\ &= \text{lb total ThOD removed/lb total substrate removed} \\ &= 32/30 \end{aligned} \tag{5.40}$$

or

$$a = a_{\text{ThOD}}(32/30) \tag{5.41}$$

Equation (5.38) written in terms of $a_{\text{ThOD}}$ by utilizing Eq. (5.41) is

$$\underbrace{(150/113)(32/30)a_{\text{ThOD}}}_{\bar{a}} + a' = 1.0$$

or

$$\underbrace{1.42a_{\text{ThOD}}}_{\bar{a}} + a' = 1.0 \tag{5.42}$$

MLVSS yield (synthesis) is obtained from Eq. (5.39).

$$\text{lb MLVSS produced} = a_{\text{ThOD}}(\text{lb total ThOD removed}) \tag{5.43}$$

Equation (5.43) may be rewritten expressing substrate removal in terms of COD, BOD, etc., by utilizing ratios ThOD/COD, ThOD/BOD, etc.:

$$\begin{aligned}\text{lb MLVSS produced} &= a_{\text{ThOD}}(\text{lb total COD removed})(\text{ThOD/COD}) \\ &= a_{\text{ThOD}}(\text{lb total BOD removed})(\text{ThOD/BOD})\end{aligned} \tag{5.44}$$

Define

$$a_{\text{COD}} = a_{\text{ThOD}}(\text{ThOD/COD}) \tag{5.45}$$

$$a_{\text{BOD}} = a_{\text{ThOD}}(\text{ThOD/BOD}) \tag{5.46}$$

Therefore

$$\begin{aligned}\text{lb MLVSS produced} &= a_{\text{COD}}(\text{lb total COD removed}) \\ &= a_{\text{BOD}}(\text{lb total BOD removed})\end{aligned} \tag{5.47}$$

No subscripts are utilized for the parameter $a$ hence. It is understood that the appropriate value of parameter $a$ is chosen to be compatible with the parameters for expressing substrate removal.

### 4.1.5. An Observation Concerning Factor 1.42

Although factor 1.42 in Eq. (5.42) is obtained in Section 4.1.4 from consideration of the specific lactose example, it is shown next that it applies to all substrates, provided the average empirical formula for the MLVSS is taken as $C_5H_7NO_2$. Consider the specific lactose example. Write Eqs. (5.20) and (5.22), multiplying the first one by a factor of 5, i.e.,

$$5(CH_2O) + 5O_2 \rightarrow 5CO_2 + 5H_2O \tag{5.48}$$

Molecular weight: $5 \times 30$ $5 \times 32$

Recall that factor 1.42 originated from [Eq. (5.42)].

$$(150/113)(32/30) = 1.42$$

or (5.49)

$$[5(30)]/113 \times 32/30 = 1.42$$

The "molecule" of substrate is defined here as a sugar group ($CH_2O$) containing *one* carbon atom, which corresponds to a "molecular weight" of 30. Notice that in Eq. (5.49), the molecular weight of substrate (30 in this case) is canceled out. For any substrate of molecular weight $M$, Eq. (5.49) is

$$5M/113 \times 32/M = (5 \times 32)/113 = 1.42 \tag{5.50}$$



Thus, Eq. (5.42) is an *approximate* equation for most substrates, the only restriction being the assumption that the average empirical formula for MLVSS is $C_5H_7NO_2$. In Section 4.1.9, it is shown that value 1.42 corresponds to lb of oxygen required to oxidize 1 lb of MLVSS during the process of endogenous respiration.

### 4.1.6. Summary

*Note*: Approximate values of the ratio between parameters for expressing oxygen demand are taken from Table 2.1.

***a. Parameter a' in Different Units (See Tabulation Below)***

| | |
|---|---|
| $a'_{\text{ThOD}}$ | lb $O_2 = a'_{\text{ThOD}}$ (lb total ThOD removed)<br>(energy)<br>where $a'_{\text{ThOD}} = a'$ = fraction of substrate removed utilized for energy production |
| $a'_{\text{COD}}$<br>(standard COD test) | lb $O_2 = a'_{\text{COD}}$ (lb total COD removed)<br>(energy)<br>where $a'_{\text{COD}} = a'(\text{ThOD/COD}) = a'(100/83) = 1.20a'$ |
| $a'_{\text{BOD}}$<br>(5-day BOD) | lb $O_2 = a'_{\text{BOD}}$ (lb total BOD removed)<br>(energy)<br>where $a'_{\text{BOD}} = a'(\text{ThOD/BOD}) = a'(100/58) = 1.72a'$ |

Relationships for other oxygen and carbon parameters studied in Chapter 2 are readily written.

***b. Parameter a in Different Units (See Tabulation Below)***

| | |
|---|---|
| $a_{\text{ThOD}}$ | lb MLVSS produced = $a_{\text{ThOD}}$(lb total ThOD removed)<br>where $a_{\text{ThOD}} = \bar{a}/1.42$; $\bar{a}$ = fraction of substrate removed utilized for synthesis |
| $a_{\text{COD}}$<br>(standard COD test) | lb MLVSS produced = $a_{\text{COD}}$(lb total COD removed)<br>where $a_{\text{COD}} = a_{\text{ThOD}}(\text{ThOD/COD}) = a_{\text{ThOD}}(100/83)$<br>$= (\bar{a}/1.42)(100/83) = 0.85\bar{a}$ |
| $a_{\text{BOD}}$<br>(5-day BOD) | lb MLVSS produced = $a_{\text{BOD}}$(lb total BOD removed)<br>where $a_{\text{BOD}} = a_{\text{ThOD}}(\text{ThOD/BOD}) = a_{\text{ThOD}}(100/58)$<br>$= (\bar{a}/1.42)(100/58) = 1.21\bar{a}$ |

Relationships for other oxygen and carbon parameters defined in Chapter 2 are readily written.

### c. *Equation (5.24) Written with Different Units for the Parameters (See Tabulation Below)*

$a'$ = fraction of the total substrate removed utilized for energy = $a'_{\text{ThOD}}$; $\bar{a}$ = fraction of total substrate removed utilized for synthesis. Then $\bar{a}+a' = 1.0$.

| | |
|---|---|
| ThOD | $1.42a_{\text{ThOD}}+a'_{\text{ThOD}} = 1.0$<br>or $1.42a_{\text{ThOD}}+a' = 1.0$ |
| COD<br>(standard COD test) | $a_{\text{ThOD}} = a_{\text{COD}}(\text{COD/ThOD}) = a_{\text{COD}}(83/100)$<br>$a'_{\text{ThOD}} = a'_{\text{COD}}(\text{COD/ThOD}) = a'_{\text{COD}}(83/100)$<br>$\therefore 1.42(83/100)a_{\text{COD}}+(83/100)a'_{\text{COD}} = 1.0$<br>$1.18a_{\text{COD}}+0.83a'_{\text{COD}} = 1.0$ |
| BOD<br>(5-day BOD) | $a_{\text{ThOD}} = a_{\text{BOD}}(\text{BOD/ThOD}) = a_{\text{BOD}}(58/100)$<br>$a' = a'_{\text{ThOD}} = a'_{\text{BOD}}(\text{BOD/ThOD}) = a'_{\text{BOD}}(58/100)$<br>$\therefore 1.42(58/100)a_{\text{BOD}}+(58/100)a'_{\text{BOD}} = 1.0$<br>$0.82a_{\text{BOD}}+0.58a'_{\text{BOD}} = 1.0$ |

Equation (5.24) is readily written in terms of other oxygen and carbon parameters defined in Chapter 2.

### 4.1.7. Design Parameters Corresponding to Endogenous Respiration: Introduction

Two design parameters, $b$ and $b'$, are defined corresponding to the endogenous respiration phase. Endogenous respiration involves oxidation of cellular matter in order to provide food for the microorganisms when the concentration of substrate has decreased considerably. It corresponds to the "cannibalistic feast" described in Section 3.1.

Assuming that the chemical formula for the MLVSS is $C_5H_7NO_2$, oxidation of cells corresponding to endogenous respiration is given by Eq. (5.51).

$$\begin{array}{lccc} & C_5H_7NO_2 + & 5O_2 & \rightarrow 5CO_2 + NH_3 + 2H_2O \\ \text{Molecular weight:} & 113 & 5\times 32 = 160 & \end{array} \quad (5.51)$$

### 4.1.8. Definition of Parameter *b* (Endogenous Respiration)

Parameter $b$ is defined as fraction of MLVSS per unit time ($\text{day}^{-1}$, $\text{hour}^{-1}$, etc.) oxidized during process of endogenous respiration. For example, a value of $b = 0.1\ \text{day}^{-1}$ means that 10% of the total lb of MLVSS present in the reactor at any time is oxidized per day. Therefore, endogenous respiration

$$b = \text{lb MLVSS oxidized/(day)(lb MLVSS in reactor)} \quad (5.52)$$

Consequently, the lb of MLVSS oxidized per day are

$$\underset{\text{(endogenous respiration)}}{\text{lb MLVSS oxidized/day}} = b(\text{lb MLVSS in reactor}) \quad (5.53)$$



MLVSS present in reactor at any time assuming steady state operation is constant, being given by

$$\text{lb MLVSS in reactor} = X_{v,a}V \quad (5.54)$$

where $X_{v,a}$ is the concentration of MLVSS, i.e., lb MLVSS per unit volume of reactor; and $V$ the reactor volume.
Thus Eqs. (5.53) and (5.54) yield

$$\underset{\text{(endogenous respiration)}}{\text{lb MLVSS oxidized/day}} = bX_{v,a}V \quad (5.55)$$

### 4.1.9. Definition of Parameter *b'*

Parameter $b'$ is defined as the lb of oxygen utilized per day per lb of MLVSS in the reactor for the process of endogenous respiration, i.e., [Eq. (5.56)]

$$b' = \text{lb } O_2/(\text{day})(\text{lb MLVSS inreactor}) \quad (5.56)$$

Thus, oxygen utilization for endogenous respiration is

$$\underset{\text{(endogenous respiration)}}{\text{lb } O_2/\text{day}} = b'(\text{lb MLVSS in reactor}) \quad (5.57)$$

or from Eq. (5.54)

$$\underset{\text{(endogenous respiration)}}{\text{lb } O_2/\text{day}} = b'X_{v,a}V \quad (5.58)$$

The approximate relationship between $b$ and $b'$ is written assuming that average empirical formula for MLVSS is $C_5H_7NO_2$, and that endogenous respiration corresponds to chemical equation (5.51). From Eqs. (5.52) and (5.56) ratio $b'/b$ is [Eq. (5.59)]

$$b'/b = \text{lb } O_2/\text{lb MLVSS oxidized} \quad (5.59)$$

From Eq. (5.51) this ratio is

$$b'/b = 1.42 \quad (5.60)$$

Consequently, it takes approximately 1.42 lb of oxygen to oxidize 1 lb of MLVSS. This value is used as an approximation for aerobic degradation of most substrates.

Whereas parameters $a$ and $a'$ are ratios [Eqs. (5.25), (5.32), (5.33), and (5.34) for $a'$; and Eqs. (5.35) and (5.39) for $a$], $b$ and $b'$ are rates. Time is not involved in the definitions of $a$ and $a'$, but it is in those of $b$ and $b'$.

## 4.2. MATERIAL BALANCE FOR DETERMINATION OF OXYGEN UTILIZATION

Knowledge of oxygen requirements to effect a specified BOD removal is necessary for specification of aeration equipment. From discussions in Sections 4.1.3 and 4.1.9 it follows that oxygen is required for two purposes:

(1) to oxidize substrate in order to provide energy requirements for cells [Eq. (5.20)] and (2) for the endogenous respiration process [Eq. (5.51)].

1. *Oxygen required for energy.* The lb of oxygen required per day are calculated from Eq. (5.31). Referring to Fig. 5.1 and symbols defined in Table 5.1,

$$\underset{\text{(for energy)}}{\text{lb } O_2/\text{day}} = a'(S_o - S_e)Q \tag{5.61}$$

Appropriate values of $a'$ compatible with parameters in which total substrate removal $(S_o - S_e)$ is expressed are utilized in Eq. (5.61).

**Example 5.1**

Calculate the oxygen required for energy.

$a'_{\text{BOD}} = 0.79$ lb $O_2$ (for energy)/lb total $BOD_5$ removed*

$S_o = 893$ mg/liter

$S_e = 40$ mg/liter

$Q = 2.04$ MGD $\quad (2.04 \times 10^6$ gal/day)

Then

$$S_o - S_e = 893 - 40 = 853 \text{ mg/liter} = S_r \quad \text{(total substrate removed)}$$

Therefore

$$S_o - S_e = S_r = 853\frac{\text{mg BOD}_r}{\text{liter liquor}} = 853\frac{\text{mg BOD}_r}{10^3\text{g liquor}}$$

$$= 853\frac{\text{g BOD}_r}{10^6\text{g liquor}} = 853 \text{ ppm} = 853 \times 10^{-6}\frac{\text{g BOD}_r}{\text{g liquor}}$$

$$= 853 \times 10^{-6}\frac{\text{lb BOD}_r}{\text{lb liquor}} = 853\frac{\text{lb BOD}_r}{\text{Mlb liquor}}$$

From Eq. (5.61),

$$\underset{\text{(for energy)}}{\text{lb } O_2/\text{day}} = 0.79\frac{\text{lb } O_2}{\text{lb BOD}_r} \times 853 \times 10^{-6}\frac{\text{lb BOD}_r}{\text{lb liquor}}$$

$$\times 2.04 \times 10^6\text{gal}\frac{\text{liquor}}{\text{day}} \times 8.34\frac{\text{lb liquor}}{\text{gal liquor}}$$

$$= 11{,}500 \text{ lb } O_2/\text{day}$$

* Experimental determination of parameter $a'$ is described in Section 6.3.2. Example 5.1 is simply an illustration of unit conversion. Value $a' = 0.79$ is determined experimentally (Example 5.5, Section 6.4).



If $S_r$ is in mg/liter and $Q$ in MGD owing to cancellation of factors $10^{-6}$ and $10^6$, Eq. (5.61) becomes Eq. (5.62).

$$\underset{\text{(for energy)}}{\text{lb } O_2/\text{day}} = a'S_rQ \times 8.34 \tag{5.62}$$

2. *Oxygen required for endogenous respiration.* Equation (5.58) is utilized for this calculation, illustrated by Example 5.2.

**Example 5.2**

Calculate the oxygen required for endogenous respiration. Let

$b' = 0.15$ lb $O_2$/(day)(lb MLVSS in reactor)*

$X_{v,a} = 300$ mg/liter (of MLVSS)

$V = 1.2$ MG $\quad (1.2 \times 10^6$ gal) (reactor volume)

By a similar procedure to that in Example 5.1 it follows that [Eq. (5.63)]

$$\underset{\text{(endogenous respiration)}}{\text{lb } O_2/\text{day}} = b'X_{v,a}V \times 8.34 \tag{5.63}$$

where $b'$ is the lb $O_2$/(day)(lb MLVSS in reactor), $X_{v,a}$ the mg/liter of MLVSS, and $V$ the reactor volume (MG).
Consequently,

$$\underset{\text{(endogenous respiration)}}{\text{lb } O_2/\text{day}} = 0.15 \times 3000 \times 1.2 \times 8.34 = 4500 \text{ lb } O_2/\text{day}$$

SUMMARY Total oxygen utilization is given by the sum of Eqs. (5.61) and (5.58) as

$$\text{lb } O_2/\text{day} = a'(S_o - S_e)Q + b'X_{v,a}V = a'S_rQ + b'x_{v,a}V \tag{5.64}$$

For Examples 5.1 and 5.2,

$$\text{lb } O_2/\text{day} = 11{,}500 + 4500 = 16{,}000 \text{ lb } O_2/\text{day}$$

## 4.3. MATERIAL BALANCE FOR DETERMINATION OF NET YIELD OF BIOLOGICAL SLUDGE (MLVSS)

From Sections 4.1.4 and 4.1.8 it follows that (1) a fraction of the substrate removed is utilized in production of MLVSS, the lb of MLVSS produced being given by Eq. (5.47), and that (2) part of the sludge produced is destroyed by oxidation (endogenous respiration), the lb of sludge oxidized being given by Eq. (5.55).

* Experimental determination of parameter $b'$ is described in Section 6.3.2. Example 5.2 is simply an illustration of unit conversion. Value $b' = 0.15$ is determined experimentally (Example 5.5, Section 6.4).

1. *Sludge produced from substrate removal.* Sludge produced in lb/day is calculated from Eq. (5.47), where total substrate removal refers to one-day production. Referring to Fig. 5.1 and symbols defined in Table 5.1,

$$\text{lb/day of MLVSS produced} = a(S_o - S_e)Q = aS_rQ \qquad (5.65)$$

Appropriate values of $a$ compatible with parameters in which total substrate removal $(S_o - S_e)$ is expressed are utilized in Eq. (5.65).

**Example 5.3**

Calculate MLVSS produced by substrate removal. Let

$a$ = 0.575 lb MLVSS produced/lb total $BOD_5$ removed*

$S_o$ = 893 mg/liter

$S_e$ = 40 mg/liter

$Q$ = 2.04 MGD $\quad (2.04 \times 10^6$ gal/day)

Conversion of units for Eq. (5.65) is similar to that for Eq. (5.61) (Example 5.1, Section 4.2). The final result is Eq. (5.66).

$$\text{lb/day MLVSS produced} = aS_rQ \times 8.34 \qquad (5.66)$$

where $S_r$ is in mg/liter and $Q$ in MGD.
Therefore,

$$\text{lb/day MLVSS produced} = 0.575(893 - 40) \times 2.04 \times 8.34$$
$$= 8342 \text{ lb/day of MLVSS}$$

2. *Sludge destroyed by endogenous respiration.* Sludge destroyed by endogenous respiration is obtained from Eq. (5.55). This calculation is illustrated by Example 5.4.

**Example 5.4**

Calculate MLVSS destroyed by endogenous respiration. Let

$b$ = 0.075 lb MLVSS oxidized/(day)(lb MLVSS in reactor) = $\text{day}^{-1}$†

$X_{v,a}$ = 3000 mg/liter

$V$ = 1.2 MG $\quad (1.2 \times 10^6$ gal; reactor volume)

* Experimental determination of parameter $a$ is described in Section 6.3.4. Example 5.3 is simply an illustration of unit conversion. Value $a = 0.575$ is determined experimentally (Example 5.5, Section 6.4).

† Experimental determination of the parameter $b$ is described in Section 6.3.4. Example 5.4 is simply an illustration of unit conversion. Value $b = 0.075$ is determined experimentally (Example 5.5, Section 6.4).



Conversion of units for Eq. (5.55) is similar to that for Eq. (5.58) (Example 5.2, Section 4.2). The final result is Eq. (5.67).

$$\text{lb MLVSS oxidized/day} = bX_{v,a}V \times 8.34 \qquad (5.67)$$

where $X_{v,a}$ is in mg/liter and $V$ in MG.
Therefore

$$\text{lb MLVSS oxidized/day} = 0.075 \times 3000 \times 1.2 \times 8.34 = 2252 \text{ lb/day of MLVSS}$$

SUMMARY Net yield of MLVSS is obtained by the difference between MLVSS produced [Eq. (5.65)] and MLVSS oxidized (endogenous respiration), given by Eq. (5.55). This net yield in lb/day is denoted as $\Delta X_v$ [Eq. (5.68)].

$$\text{lb MLVSS/day (net yield)} = \Delta X_v = a(S_o - S_e)Q - bX_{v,a}V = aS_rQ - bX_{v,a}V \qquad (5.68)$$

For examples 5.3 and 5.4

$$\Delta X_v = 8342 - 2252 = 6090 \text{ lb/day}$$

## 4.4. TOTAL SLUDGE YIELD

So far, only the yield of biological sludge (MLVSS) has been considered. Now, examine the diagram for the reactor system in Fig. 5.1. The fresh feed may contain nonvolatile suspended solids (NVSS). Let $X_{NV,F}$ be the concentration (mg/liter) of these NVSS.

Reactor contents are under conditions of complete mixing, therefore no settling of MLNVSS (or MLVSS) takes place. Consequently, concentration of NVSS in reactor effluent is the same as that in combined feed ($X_{NV,o}$). In the secondary clarifier, however, part of the NVSS as well as most of VSS settles. Let $X_{NV,u}$ be the concentration of NVSS in underflow from the clarifier (same as in wastage $Q$ and recycle $Q_R$). Concentration of NVSS in net effluent from clarifier ($Q'$) is $X_{NV,e}$.

Wastage of sludge corresponds to

1. Net yield of biological sludge (MLVSS) from the reactor. This is $\Delta X_v$ [Eq. (5.68)]. Since the reactor operates at steady state, this wastage is equal to net yield of MLVSS, so that total lb of MLVSS in the reactor remain the same at all times. In addition, wastage includes volatile solids entering with fresh feed ($Q_F X_{V,F}$), as seen from an overall balance of volatile solids (loop – – – in Fig. 5.1). Therefore, total wastage of MLVSS is shown in Eq. (5.69) [utilizing Eq. (5.68) for $\Delta X_v$].

$$\Delta X_v + Q_F X_{V,F} = a(S_o - S_e)Q - bX_{v,a}V + Q_F X_{V,F} \qquad (5.69)$$

2. Settled NVSS denoted as $\Delta X_{NV}$ (lb/day). This value is determined by

an overall material balance for NVSS over loop ---- in Fig. 5.1.

NVSS, IN: $Q_F X_{NV,F}$

NVSS, OUT: $Q'X_{NV,e} + Q''X_{NV,u} = Q'X_{NV,e} + \Delta X_{NV}$ (since $\Delta X_{NV} = Q''X_{NV,u}$) (5.70)

Thus the overall balance is [Eq. (5.71)]

$$Q_F X_{NV,F} = Q'X_{NV,e} + \Delta X_{NV}$$

$$\therefore \Delta X_{NV} = Q_F X_{NV,F} - Q'X_{NV,e} \tag{5.71}$$

Eliminating $Q'$ and utilizing Eq. (5.2),

$$\Delta X_{NV} = Q''X_{NV,u} = Q_F X_{NV,F} - (Q_F - Q'')X_{NV,e} = Q_F(X_{NV,F} - X_{NV,e}) + Q''X_{NV,e} \tag{5.72}$$

Substitution of $\Delta X_v$ and $\Delta X_{NV}$ in Eq. (5.1) by their values given by Eqs. (5.68) and (5.72) yields total sludge yield $\Delta X_t$ [Eq. (5.73)].

$$\Delta X_t = a(S_o - S_e)Q - bX_{v,a}V + Q_F X_{V,F} + Q_F(X_{NV,F} - X_{NV,e}) + Q''X_{NV,e} \tag{5.73}$$

where

$a(S_o - S_e)Q - bX_{v,a}V = \Delta X_v$ = net yield of MLVSS [Eq. (5.68)]

$Q_F X_{V,F}$ = MLVSS in fresh feed

$Q_F(X_{NV,F} - X_{NV,e}) + Q''X_{NV,e} = \Delta X_{NV}$ = net yield of sludge due to settling NVSS from influent [Eq. (5.72)]

## 4.5. MATERIAL BALANCES FOR $X_{NV,o}$ AND $X_{v,o}$

The value of $X_{NV,o}$, i.e., concentration of NVSS in combined feed, is established by a material balance around the junction of the fresh feed with the recycle to form combined feed (Fig. 5.1, loop -·-·-).

$$\text{NVSS, IN} = Q_F X_{NV,F} + Q_R X_{NV,u}$$

$$\text{NVSS, OUT} = QX_{NV,o}$$

Then

$$Q_F X_{NV,F} + Q_R X_{NV,u} = QX_{NV,o}$$

Utilizing Eqs. (5.4) and (5.5) and solving for $X_{NV,o}$,

$$X_{NV,o} = (X_{NV,F} + rX_{NV,u})/(1+r) \tag{5.74}$$

A similar material balance is written for $X_{v,o}$, the concentration of VSS in combined feed. Final result is

$$X_{v,o} = (X_{V,F} + rX_{v,u})/(1+r) \tag{5.75}$$



## 4.6. TYPICAL VALUES OF AEROBIC BIOLOGICAL WASTEWATER TREATMENT PARAMETERS FOR DIFFERENT TYPES OF WASTEWATERS

Typical values of these parameters are presented in Table 5.2.

**TABLE 5.2**
**Aerobic Biological Waste—Treatment Parameters[a, b]**

| Wastewater | $a$ | $a'$ | $b$ | $b'$[c] | $k$ |
|---|---|---|---|---|---|
| Domestic | 0.73 | 0.52 | 0.075 | 0.106 | 0.017–0.03 |
| Refinery | 0.49–0.62 | 0.40–0.77 | 0.10–0.16 | 0.142–0.227 | 0.074 |
| Chemical and petrochemical | 0.31–0.72 | 0.31–0.76 | 0.05–0.18 | 0.071–0.255 | 0.0029–0.018 |
| Brewery | 0.56 | 0.48 | 0.10 | 0.142 | — |
| Pharmaceutical | 0.72–0.77 | 0.46 | — | — | 0.018 |
| Kraft pulping and bleaching | 0.5 | 0.65–0.8 | 0.08 | 0.114 | — |

[a] Adapted from Ref. [2].
[b] Units: $a$, lb MLVSS produced/lb total $BOD_5$ removed; $b$, lb MLVSS oxidized/(day)(lb MLVSS in reactor) = $day^{-1}$; $a'$, lb $O_2$ (for energy)/lb total $BOD_5$ removed; $b'$, lb $O_2$/(day)(lb MLVSS in reactor) = $day^{-1}$; $k$, $day^{-1}$.
[c] Values of $b'$ estimated from $b' = 1.42b$.

# 5. Relationship for Optimum Settling Conditions of Sludge

For adequate operation of the activated sludge process, MLVSS in the reactor effluent should be readily separated in the secondary clarifier. The condition occurring when sludge is light and fluffy and thus difficult to settle is termed bulking. Bulky sludge flakes over separating weirs and comes out with the secondary clarifier effluent. Since concentration of substrate in the effluent is small, there is not enough food material to sustain the growth of the microorganisms which constitute the sludge. Therefore the microorganisms are driven to endogenous respiration. Owing to the consumption of oxygen for endogenous respiration, the effluent has a relatively high BOD, which is undesirable.

Settling characteristics of sludge are evaluated from sedimentation tests performed in the laboratory. For this evaluation two parameters are utilized.

1. *Zone settling velocity* (*ZSV*). This parameter and its experimental determination are discussed in Chapter 3, Section 3.6. An easily settling sludge has a high ZSV of about 20 ft/hr.

2. *Sludge volume index (SVI).* Sludge volume index is defined as volume (in $cm^3$) occupied by 1 g of dry sludge solids after settling for 30 min. The smaller the SVI, the easier is the settling of the sludge.

Several authors have correlated settling characteristics of sludge (in terms of ZSV or SVI) with a parameter designated as food to microorganism ratio (hence denoted as $F/M$). This parameter is defined as [Eq. (5.76)]

$$F/M = \text{lb of substrate in influent/(day)(lb MLVSS in reactor)} \tag{5.76}$$

Values of $F$ and $M$ are given by

$$F = (QS_o) \times 8.34 \quad \text{(lb/day)} \tag{5.77}$$

$$M = (X_{v,a}V) \times 8.34 \quad \text{(lb)} \tag{5.78}$$

where $Q$ is in MGD and ($S_o$, $X_{v,a}$) in mg/liter. Therefore

$$F/M = QS_o/X_{v,a}V \tag{5.79}$$

Since $V/Q = t$ = residence time,

$$F/M = S_o/X_{v,a}t \quad (\text{day}^{-1}) \tag{5.80}$$

In order to arrive at correlations for settling characteristics of a sludge, a series of bench scale continuous reactors are operated, each at a selected $F/M$ ratio. Sludge obtained in each reactor is subjected to settling tests (ZSV and SVI). If these two parameters, which are a measure of the ability of the sludge to settle, are plotted vs. the corresponding $F/M$ ratios, curves like the ones shown in Fig. 5.7 are obtained.

Since for optimum settling the sludge should have a high ZSV and a low SVI, the optimum $F/M$ ratio as indicated in Fig. 5.7 corresponds to the maximum for the ZSV curve and the minimum for the SVI curve. For most wastewaters this optimum value of the $F/M$ ratio falls between the following limits [Eq. (5.81)]:

$$0.6 > F/M > 0.3 \tag{5.81}$$

where $F/M$ is expressed in lb $BOD_5$ influent/(day)(lb MLVSS). An explanation for the correlation between $F/M$ ratio and sedimentation characteristics of the sludge is given below.

1. At low $F/M$ ratios (e.g., below $F/M = 0.3$) the amount of food (substrate) present in the system is insufficient to maintain the growth of the microorganisms. Therefore, they are driven to endogenous respiration. A typical bacterial cell is shown in Fig. 5.8. Cytoplasmic material is rich in proteins and ribonucleic acid (RNA), and it is the main portion of the cell which is metabolized during the process of endogenous respiration. The residue left from endogenous metabolism is constituted mainly by cell capsules, which are very light and resist sedimentation. This is why at low $F/M$ ratios,



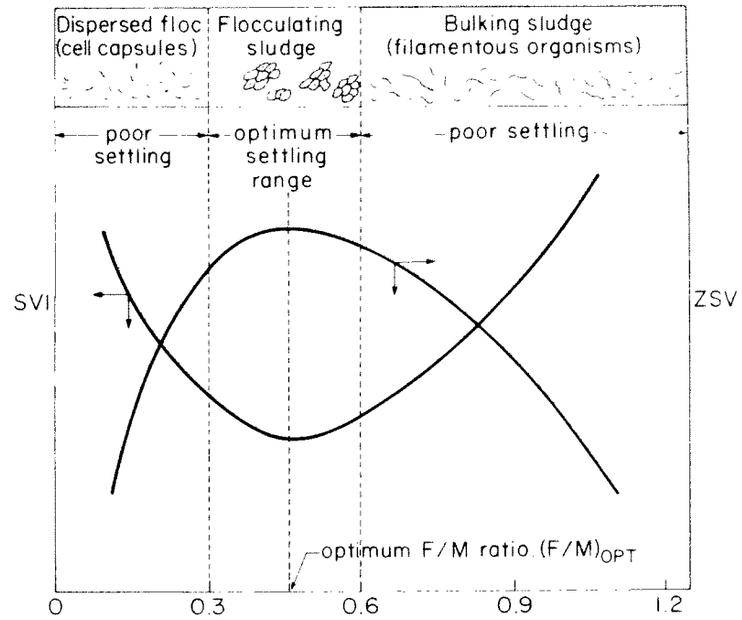


*Fig. 5.7. Typical correlation of SVI and ZSV with F/M ratio.*

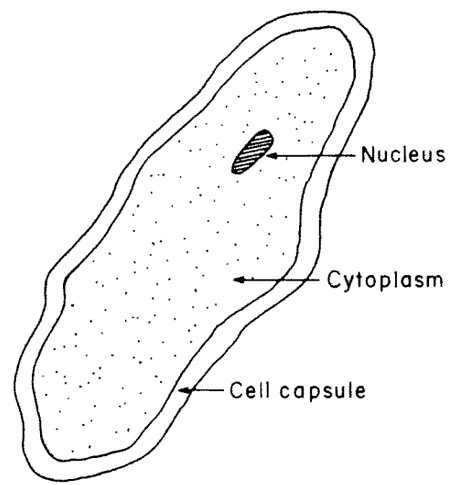


*Fig. 5.8. Typical bacterial cell.*

the sludge has poor settling characteristics. Sludge obtained under these conditions is referred to as dispersed floc, and a microscopic view of it is shown in Fig. 5.7 for the region of low $F/M$ ratios.

2. At high $F/M$ ratios (e.g., $F/M > 0.6$) there is predominance of a type of microorganism which is filamentous in nature (*Sphaerotilus*). This type of growth does not settle well, remaining in suspension almost indefinitely. Sludge under these conditions is referred to as a bulking sludge.

3. At values of the $F/M$ ratio between these two extremes, sludge with good settling characteristics is obtained. Sludge under these conditions is referred to as flocculating sludge.

From Eq. (5.80) the residence time $t$ to yield an optimum flocculating sludge is obtained. Written for the optimum $F/M$ ratio as determined from Fig. 5.7, Eq. (5.80) is

$$(F/M)_{\text{OPT}} = S_o/X_{v,a}t \tag{5.82}$$

Solving for $t$,

$$t = S_o/[X_{v,a}(F/M)_{\text{OPT}}] \tag{5.83}$$

The geometry of the system and the manner in which wastewater is fed to the aerator have an effect on flocculating characteristics of the sludge. For example, if the aerator is a long rectangular tank with relatively poor mixing, MLVSS is initially contacted at the feed end with entering sewage, and therefore a high $F/M$ ratio prevails at the entrance. Filamentous growth developed under these conditions persists throughout the aeration period, and sludge with poor settling characteristics is obtained (Fig. 5.9). The same situation

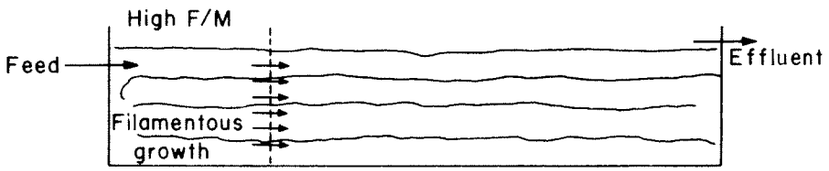


***Fig. 5.9. Effect of geometry in settling characteristics of MLVSS (plug flow model).***

occurs in a batch reactor, since a high $F/M$ ratio prevails at the start of the operation. The reactor depicted in Fig. 5.9 is the plug flow continuous reactor. A general discussion of the kinetics of a continuous treatment system (plug flow, complete mix, and arbitrary flow reactors) is presented in Section 10.

If there is complete mixing in the system, the $F/M$ ratio is uniform throughout, possibly falling within the optimum range. Under steady state and complete mix conditions, sludge is always in contact with a BOD concentration equal to that in the effluent. Therefore a dense sludge is likely to be obtained.



It is important to obtain experimentally the graph in Fig. 5.7 for the specific substrate under study, since considerable variation occurs depending on substrate characteristics. Substrates which are easily degradable (e.g., soluble sugars) become immediately available as food to the microorganisms, and therefore the result is a fast growth response. On the other hand, complex organic substrates (e.g., wastewaters from petroleum and petrochemical plants) must undergo chemical breakdown before being available as food to the microorganisms, growth response being therefore slower.

# 6. Experimental Determination of Parameters Needed for Design of Aerobic Biological Reactors

## 6.1. BENCH SCALE CONTINUOUS REACTORS

A bench scale continuous reactor utilized for these determinations is described in this section. Parameters to be determined are defined in Sections 4.1.2 to 4.1.9, i.e., for kinetic relationship: $k$; for material balance relationships: $a$, $a'$, $b$, and $b'$. A diagram of the continuous flow reactor is shown in Fig. 5.10. This unit is designed and built by Bio-Development Associates, Austin, Texas. The reactor is made of plexiglass and divided into two sections: the aeration and settling chambers. These simulate the reactor and the secondary clarifier for an actual plant.

Capacity of the aeration chamber is approximately 7 liters. Air is supplied as indicated in the diagram. Bubbling air keeps the contents of the aeration chamber in a completely mixed condition. Wastewater is fed continuously from a constant head feed reservoir by means of a Sigmamotor pump, and overflows continuously into the effluent bottle. The aeration and sedimentation chambers are separated by a sliding baffle which can be completely removed if desired.

Start-up is performed by placing in the aeration chamber a seed of domestic activated sludge collected from an operating plant, and gradually acclimating it to the wastewater under study. For wastewaters of industrial origin containing compounds which are toxic to the microorganisms, mixtures of industrial wastewater and domestic sewage are fed to the reactor with a gradually increased proportion of industrial wastewater. Eventually, feed is 100% industrial wastewater without deleterious effects on the microorganisms.

Flow rate is varied by proper setting of the Sigmamotor pump, and by utilizing different internal diameters for the Tygon tubing. A Sigmamotor pump operates by "squeezing" the wastewater through the Tygon tubing by means of mechanical "fingers," the speed of which is set. One pump promotes wastewater flow through several reactor units in parallel, each one provided

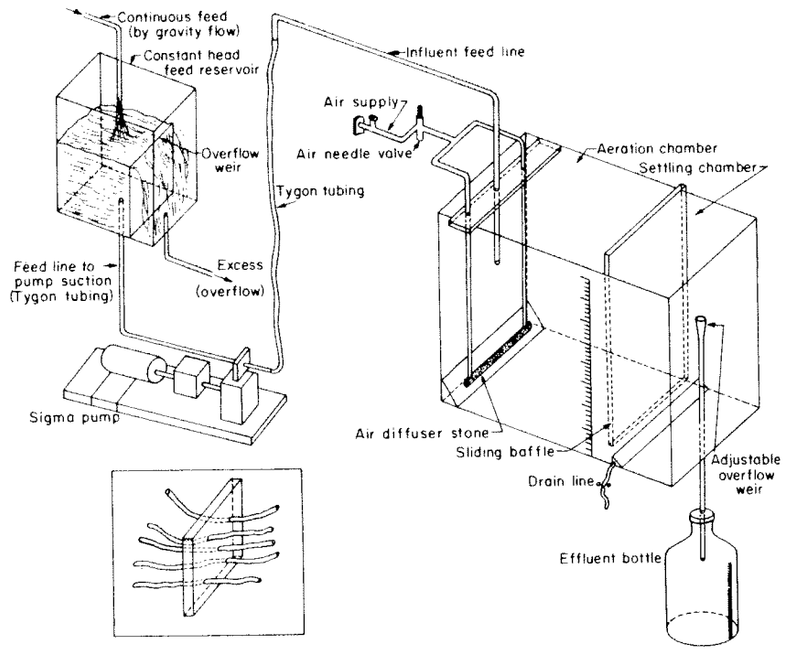


***Fig. 5.10. Continuous flow reactor (bench scale model). Insert: detail of Sigma pump setup for operation of five reactors in parallel.***

with its own Tygon feed line [Fig. 5.10 (insert)]. The "fingers" simultaneously squeeze these several Tygon tubings, promoting different flow rates for each line depending on the internal diameter of each tubing.*

Flow rates are determined by calibration, either weighing or measuring the volume of effluent obtained during a timed period corresponding to a selected settling of the pump and a chosen internal diameter of tubing. Flow rates are reproducible within less than 1% fluctuation.

Flow rates vary considerably, e.g., from 350 down to about 1.0 liter/day. For an aerator chamber volume of 7.0 liters, these rates correspond to residence times of

$$Q = 350 \text{ liters/day}, \quad t = V/Q = 7/(350/24) = 0.48 \text{ hr} \approx 30 \text{ min}$$

$$Q = 1 \text{ liter/day}, \quad t = V/Q = 7/1 = 7 \text{ days}$$

As the section of Tygon tubing subjected to this continuous squeezing

* Sigmamotor pump model T-8 (manufactured by Sigmamotor Inc., Houston, Texas) an be used to operate five units in parallel.



action wears out, it softens and flow rates change. It is advisable to slide the tubing along at periodic intervals, so that a new section of it becomes exposed to the squeezing action. Frequent calibration is performed to ensure confidence in the results. Tubing is replaced after it is worn out.

The main difference in operating principle between this bench scale reactor and the one in plant scale (Fig. 5.1) is that no controlled recycle of sludge is provided in the bench scale unit. Sludge is returned to aeration chamber from the settling chamber through the opening between the baffle and the bottom of the unit. This rate of return cannot be controlled. It is desirable to maintain the concentration of MLVSS in the aeration chamber approximately constant (at a selected value usually between 2000 and 3000 mg/liter). In order to achieve this constant MLVSS concentration, the procedure is

1. Determine periodically the MLVSS concentration in the aerator liquor from samples withdrawn through the drain line.

2. Withdraw calculated weights of MLVSS in order to keep this concentration at the selected value for a given experiment. For a reactor operating with MLVSS under endogenous respiration conditions, it is necessary to add sludge instead of withdrawing it, in order to keep a constant MLVSS concentration.

When the sliding baffle is inserted, the bench scale reactor is utilized to simulate the activated sludge unit as described. By removing the sliding baffle, simulation of an aerated lagoon is obtained (Chapter 6, Section 5).

## 6.2. EXPERIMENTAL PROCEDURE

Each experiment requires 2–4 weeks before steady state conditions are achieved. For this reason it is convenient to operate simultaneously four or five reactors in parallel.

Steps in the experimental procedure are [3]

1. Each unit is filled with seed sludge up to a predetermined volume. Dilution is made with wastewater in order to obtain a MLVSS concentration of 2000–3000 mg/liter.

2. Air is turned on and contents of the aeration chamber are completely mixed by the turbulence thus produced. The sliding baffle is adjusted to leave an opening of $\frac{1}{4}$ to $\frac{1}{2}$ in. at the bottom. During operation of the reactor, further baffle adjustments are made in order to provide a desired blanket height of sludge in the settling chamber and an interchange of sludge between the two chambers (Fig. 5.11).

3. Start the Sigmamotor pump at a flow rate necessary to obtain the desired residence time in the aeration chamber. Acclimation of sludge, if required, is performed as previously described.

4. Operate the reactor until steady state conditions are achieved. Attainment of steady state is assumed when two criteria are satisfied: (a) oxygen

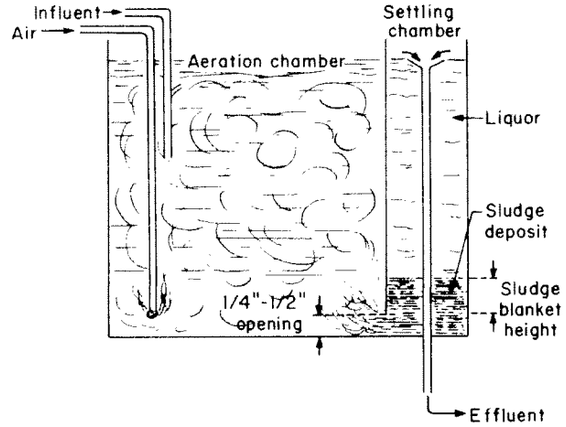


*Fig. 5.11. Side view (section) of aeration and settling chambers.*

uptake rate of reactor contents remains unchanged (determination of oxygen uptake rate is described in Section 6.3.3) and (b) BOD of effluent becomes stable.

5. Concentration of MLVSS is measured daily and adjusted to a nearly constant value for the duration of an experiment. To check net increase of MLVSS, plug overflow weir, raise baffle, and withdraw a sample from mixed tank contents. If $V_t$ is the total volume (aeration chamber plus settling chamber) and two determinations of MLVSS are made, e.g., 24 hr apart yielding values $X_1$ and $X_2$, respectively, the net increase of MLVSS is

$$\Delta X_v = V_t X_2 - V_t X_1 = V_t(X_2 - X_1) \qquad \text{(24-hr growth)} \qquad (5.84)$$

Values $(X_1, X_2)$ represent "averaged" concentrations of MLVSS for the total volume of the tank, since the baffle has been raised and the contents of aeration and sedimentation chambers mixed. Essentially, no sludge growth occurs in the sedimentation chamber because there is no direct aeration there. Therefore, the value of $\Delta X_v$ calculated from Eq. (5.84) represents the net growth occurring in the aerator.

For application of Eq. (5.68), it is recommended to take $X_{v,a}$ as the MLVSS concentration determined after the baffle is raised. This may seem controversial since in Eq. (5.68) $X_{v,a}$ stands for MLVSS concentration in aeration chamber during the operation ($V$ is the volume of aeration chamber). The concentration of MLVSS in the sedimentation chamber is probably different from that in the aerator. At the bottom of the sedimentation chamber there is a sludge blanket of very high MLVSS concentration, and at the top a supernatant liquid with negligible MLVSS concentration. After the baffle is raised,



this heterogeneous mass in the sedimentation chamber is mixed with the contents of the aerator. The whole volume is thoroughly mixed by the bubbling air before the sample is taken. The designers of this laboratory reactor claim that there is no significant difference between MLVSS concentration in the aeration chamber during operation and that in the whole mixed content of the two chambers. In any event, it is practically impossible to withdraw representative samples from the aeration chamber during the operation for analysis of $X_{v,a}$. Recall also that the volume of the sedimentation chamber is much smaller than that of the aeration chamber (ratio of about 3/7). Therefore, MLVSS concentration in the mixed contents of the two chambers is not too different from that of the aeration chamber during operation.

6. Once steady state operation is attained, the sampling schedule presented in Table 5.3 is followed.

**TABLE 5.3**
**Sampling Schedule [3]**

| Analysis | Frequency | Raw waste[a] | Mixed liquor[b] | Effluent[c] |
|---|---|---|---|---|
| 1. COD, BOD, or TOC (mg/liter) (filtered and unfiltered composite samples) | 3/week | x ($S_o$) | — | x ($S_e$) |
| 2. pH | daily | x | x | x |
| 3. SS, MLVSS (mg/liter) (also determine sludge settling curves and sludge volume index of mixed liquor at the end of test run) | 3/week | — | x ($X_{v,a}$) | x (keep low) |
| 4. Dissolved oxygen (DO) (mg/liter) | daily | — | x | — |
| 5. Oxygen uptake rate | 3/week | — | x | — |
| 6. Microscopic analysis (gram stain) | 1/week | — | x | — |
| 7. Color, turbidity | 3/week | — | — | x |
| 8. Significant ions, compounds | 3/week | x | — | x |

[a] Sample withdrawn from influent feed line or raw waste containers.
[b] Sample withdrawn from the unbaffled tank.
[c] Sample withdrawn from effluent bottle.

## 6.3. CALCULATION OF DESIGN PARAMETERS

Calculation of parameters $k$, $a$, $a'$, $b$, and $b'$ is made from obtained data. Procedure is described in Sections 6.3.1–6.3.4.

### 6.3.1. Determination of Substrate Removal Rate ($k$)

This determination, based on Eq. (5.18) or Eq. (5.19), is described in Section 3.2.

### 6.3.2. Determination of Oxygen Utilization Parameters $a'$ and $b'$

This determination is based on Eq. (5.64) in which the left-hand member is written as $R_r V$, i.e.,

$$R_r V = a'(S_o - S_e)Q + b' X_{v,a} V \tag{5.85}$$

where $R_r$ is the oxygen uptake rate, i.e., oxygen utilized per day per unit volume of reactor; and $V$ the reactor volume.

Experimental determination of $R_r$ is discussed in Section 6.3.3. Dividing Eq. (5.85) by $X_{v,a}V$ and letting $V/Q = t$ (residence time) yields

$$R_r/X_{v,a} = a'[(S_o - S_e)/X_{v,a}t] + b' \tag{5.86}$$

Equation (5.86) is the basic relationship for determination of oxygen utilization parameters $a'$ and $b'$. Notice the presence of term $(S_o - S_e)/X_{v,a}t$ (substrate removal rate), which also occurs in Eqs. (5.18) and (5.19) for determination of $k$.

Units for $R_r$ obtained from laboratory scale determinations are metric, i.e., mg $O_2$/(day)(liter). Since

mg $O_2$/liter liquor = lb $O_2$/Mlb liquor (Section 4.2, Example 5.1)

then

$R_r$ = lb $O_2$/(day)(Mlb liquor)

Similarly, for $X_{v,a}$

$X_{v,a}$ = mg MLVSS/liter liquor = lb MLVSS/Mlb liquor

Therefore in Eq. (5.86)

$$R_r/X_{v,a} = \frac{\text{lb } O_2/(\text{day})(\text{Mlb liquor})}{\text{lb MLVSS/Mlb liquor}} = \text{lb } O_2/(\text{day})(\text{lb MLVSS})$$

Thus $R_r/X_{v,a}$ is a measure of utilization of oxygen per day and per lb of biological sludge present in the reactor.

As shown in Section 3.2,

$(S_o - S_e)/X_{v,a}t$ = lb BOD removed/(day)(lb MLVSS)

According to Eq. (5.86) a plot of $R_r/X_{v,a}$ vs. $(S_o - S_e)/X_{v,a}t$ yields a straight line from the slope and intercept of which oxygen utilization parameters $a'$ and $b'$ are obtained. A typical plot is shown in Fig. 5.16, and a numerical illustration of its construction from laboratory data is presented in Example 5.5 (Section 6.4).



### 6.3.3. Experimental Determination of the Oxygen Uptake Rate ($R_r$)

Possibly the simplest way to determine the oxygen uptake rate is by galvanic cell oxygen measurements. This is the only method described in this section. Other methods are polarographic and Warburg techniques and off-gas analysis. Of all these methods, galvanic cell measurement is the simplest, and its accuracy is usually adequate. The apparatus for this measurement is the dissolved oxygen analyzer (DO analyzer) described in Chapter 2 (Section 2.3.1) and shown in Fig. 2.4.

Experimental technique for measuring oxygen uptake rate ($R_r$) is [6]

1. Fill BOD bottle with aerated mixed liquor from test solution.
2. Insert probe into bottle, allowing displaced liquid to overflow. Care is taken to prevent accumulation of air bubbles inside bottle.
3. Mix the contents using a magnetic stirring apparatus.
4. Record galvonometer readings at various time intervals, usually every 30 sec.
5. Correct readings based on a predetermined sensitivity factor (for details refer to [6]), and plot dissolved oxygen level (ordinate) vs. time (abscissa) (Fig. 5.12).

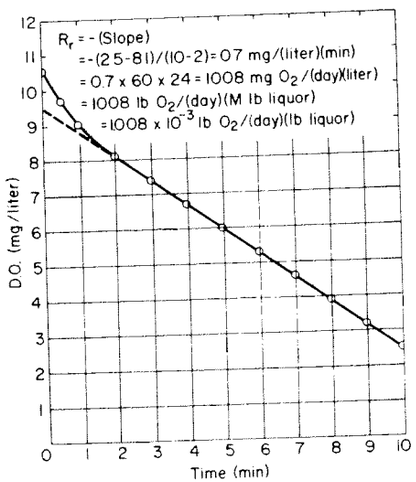


*Fig. 5.12. Determination of oxygen uptake rate.*

6. In Fig. 5.12, the slope of the line is oxygen uptake rate in mg/(liter)(min). A specific uptake rate ($R_r/X_{v,a}$) is then determined by dividing this value by MLVSS concentration in the test sample. In Fig. 5.12, the first data points immediately after $t = 0$ are *not* to be taken into account in evaluating the slope. The higher slope of this section of the line is due to loss of entrained air from the liquor. After a few minutes the slope becomes stabilized, and it is taken as the uptake rate.

A temperature correction available from nomographs furnished by the manufacturer is applied to the readings. Probe readings are inaccurate at DO concentrations below 0.5 mg/liter. Transfer of the mixed liquor from the reactor to the DO analyzer bottle should be rapid, and the test started as soon as possible following sample withdrawal. If oxygen depletion is too rapid, the sample is diluted in order to reduce MLVSS concentration. It is advisable to calibrate the probe in a sample of water similar to that in which the DO analyzer is used, in order to eliminate errors due to the salt effect.

### 6.3.4. Determination of Parameters for Sludge Yield (*a* and *b*)

Determination of parameters $a$ and $b$ is based on Eq. (5.73). For the bench scale reactor there is no recycle of sludge, contrary to what happens for the reactor in Fig. 5.1, for which Eq. (5.73) is written.

A simplified diagram of the bench scale reactor is shown in Fig. 5.13. By comparing Fig. 5.13 with Fig. 5.1, terms in Eq. (5.73) are modified for application to the laboratory unit.

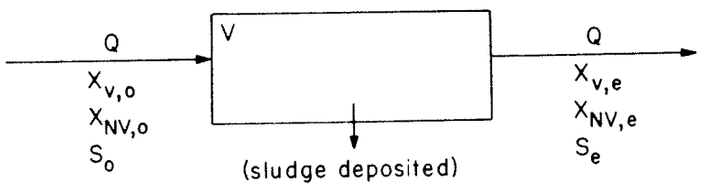


***Fig. 5.13. Simplified diagram of the bench scale continuous reactor.***

| Fig. 5.1 | Fig. 5.13 |
|---|---|
| $Q_F$ | $Q$ |
| $Q''$ | zero |
| $X_{V,F}$ | $X_{v,o}$ |
| $X_{NV,F}$ | $X_{NV,o}$ |



Therefore, Eq. (5.73) for the laboratory reactor becomes

$$\Delta X_t = \underbrace{a(S_o - S_e)Q - bX_{v,a}V}_{\Delta X_v} + QX_{v,o} + \underbrace{Q(X_{NV,o} - X_{NV,e})}_{\Delta X_{NV}} \tag{5.87}$$

Equation (5.87) is rearranged as

$$\Delta X_t - Q(X_{NV,o} - X_{NV,e} + X_{v,o}) = \Delta X_v = a(S_o - S_e)Q - bX_{v,a}V \tag{5.88}$$

Dividing through by $X_{v,a}\,V$ and noticing that $V/Q = t$ (residence time),

$$\frac{\Delta X_t/V - (X_{NV,o} - X_{NV,e} + X_{v,o})/t}{X_{v,a}} = \frac{(\Delta X_v/V)}{X_{v,a}} = a[(S_o - S_e)/X_{v,a}t] - b \tag{5.89}$$

In the numerator of the left-hand member of Eq. (5.89), term $\Delta X_t/V$ equals the net yield of total sludge per unit volume [i.e., mg total sludge/(day)(liter)]. Term $\Delta X_v/V$ corresponds to the net yield of MLVSS per unit volume. If concentrations of NVSS and MLVSS in the influent are negligible (i.e., $X_{NV,o} \approx X_{NV,e} \approx X_{v,o} \approx 0$), this equation reduces to

$$\frac{\Delta X_t/V}{X_{v,a}} = \frac{\Delta X_v/V}{X_{v,a}} = a[(S_o - S_e)/X_{v,a}t] - b \tag{5.90}$$

Equation (5.89) [or Eq. (5.90)] is the basic relationship for determination of sludge yield parameters $a$ and $b$. Notice again the presence of term $(S_o - S_e)/X_{v,a}t$ (substrate removal rate), which also occurred in Eqs. (5.18), (5.19), and (5.86) for determination of parameters $k$, $a'$, and $b'$.

*Note on units for Eq. (5.89) [or Eq. (5.90)]*: From laboratory determinations, the value of $\Delta X_t/V$ is obtained in metric units, i.e., $\Delta X_t/V$ = mg total sludge yield/(day)(liter of liquor). From similar considerations as those for $R_r$ (Section 6.3.2), it follows that this value is numerically equal to that expressed in English units, i.e., $\Delta X_t/V$ = lb sludge yield/(day)(Mlb liquor).

Therefore term $(\Delta X_t/V)/X_{v,a}$ in English units is

$$\frac{\Delta X_t/V}{X_{v,a}} = \frac{\text{lb total sludge yield/(day)(Mlb liquor)}}{\text{lb MLVSS/Mlb liquor}}$$

$$= \text{lb total sludge yield/(day)(lb MLVSS)}$$

Similarly,

$$\frac{\Delta X_t/V - (X_{NV,o} - X_{NV,e} + X_{v,o})/t}{X_{v,a}} = \text{lb MLVSS yield/(day)(lb MLVSS)}$$

According to Eq. (5.89) [or Eq. (5.90)] a plot of (accounting for presence of NVSS)

$$\left[\frac{\Delta X_t/V - (X_{NV,o} - X_{NV,e} + X_{v,o})/t}{X_{v,a}} = \frac{\Delta X_v/V}{X_{v,a}}\right] \text{ vs. } (S_o - S_e)/X_{v,a}t$$

or simply (if NVSS is negligible)

$$\frac{\Delta X_v/V}{X_{v,a}} \text{ vs. } (S_o - S_e)/X_{v,a}t$$

yields a straight line from the slope and intercept of which design parameters $a$ and $b$ are obtained. A typical plot is shown in Fig. 5.17, and its construction from laboratory data is illustrated in Section 6.4, Example 5.5.

The abscissa intercept in Fig. 5.17 corresponds to a zero value for the ordinate. This occurs for a condition of net zero yield of MLVSS, i.e., $\Delta X_v = 0$. Referring to Eq. (5.68), for $\Delta X_v = 0$ it follows that production of MLVSS by synthesis, i.e., $a(S_o - S_e)Q$, is exactly balanced by loss of MLVSS oxidized by endogenous respiration, i.e., $bX_{v,a}V$. Therefore

$$a(S_o - S_e)Q = bX_{v,a}V$$

Thus, the length of abscissa intercept is $(S_o - S_e)/X_{v,a}t = b/a$, as indicated in Fig. 5.14.

In summary, the most important information derived from bench scale studies using this laboratory reactor is the organic removal capacity of an acclimated biological sludge receiving a predefined wastewater. Full scale plants operating on design criteria developed using this reactor produce an effluent which approximates the predicted quality. Moreover, oxygen utilization rates are scaled up with relative accuracy from bench scale reactors to full scale units. There is some difficulty, however, in scaling up and applying coefficients $a$ and $b$ developed from bench scale reactors to a full scale unit because of limitations due to low accuracy of the VSS test, and the difficulty of establishing a solids balance in small scale simulation studies. Using larger reactors of pilot-plant scale enhances the accuracy of these coefficients. Fortunately, the accuracy of coefficients $a$ and $b$ is less important for the designer than those for removal rate constant ($k$) and oxygen demand coefficients ($a', b'$).

## 6.4. NUMERICAL EXAMPLES: DETERMINATION OF DESIGN PARAMETERS FOR AN ACTIVATED SLUDGE SYSTEM

### Example 5.5

An industrial plant is considering an activated sludge system for treatment of their wastewaters. Preliminary tests are performed in laboratory scale continuous reactors (Fig. 5.10). The volume of the aeration chamber in laboratory reactors is 7 liters. Four reactors are operated in parallel until steady state conditions are obtained. Data taken are presented in Table 5.4.

The influent contains an average of 30 mg/liter of NVSS. In effluent, con-



**TABLE 5.4**
**Laboratory Data**

| Reactor no. | Influent average concentration $BOD_5$ $S_o$ (mg/liter) | Effluent average concentration $BOD_5$ $S_e$ (mg/liter) | Average MLVSS concentration $X_{v,a}$ (mg/liter) | Flow rate $Q$ (liter/day) | Oxygen uptake rate $R_r$ [mg $O_2$/(liter)(day)] | Sludge yield $\Delta X_v/V$ [mg sludge/(liter)(day)] | Sludge volume index SVI |
|---|---|---|---|---|---|---|---|
| 1 | 880 | 100 | 3100 | 40.00 | 4025 | 2387 | 75.0 |
| 2 | 870 | 50 | 2800 | 14.90 | 1800 | 806 | 61.0 |
| 3 | 870 | 30 | 3000 | 8.75 | 1292 | 387 | 64.0 |
| 4 | 860 | 20 | 2900 | 3.50 | 780 | 295 | 69.0 |

or simply (if NVSS is negligible)

$$\frac{\Delta X_v/V}{X_{v,a}} \text{ vs. } (S_o - S_e)/X_{v,a}t$$

yields a straight line from the slope and intercept of which design parameters $a$ and $b$ are obtained. A typical plot is shown in Fig. 5.17, and its construction from laboratory data is illustrated in Section 6.4, Example 5.5.

The abscissa intercept in Fig. 5.17 corresponds to a zero value for the ordinate. This occurs for a condition of net zero yield of MLVSS, i.e., $\Delta X_v = 0$. Referring to Eq. (5.68), for $\Delta X_v = 0$ it follows that production of MLVSS by synthesis, i.e., $a(S_o - S_e)Q$, is exactly balanced by loss of MLVSS oxidized by endogenous respiration, i.e., $bX_{v,a}V$. Therefore

$$a(S_o - S_e)Q = bX_{v,a}V$$

Thus, the length of abscissa intercept is $(S_o - S_e)/X_{v,a}t = b/a$, as indicated in Fig. 5.14.

In summary, the most important information derived from bench scale studies using this laboratory reactor is the organic removal capacity of an acclimated biological sludge receiving a predefined wastewater. Full scale plants operating on design criteria developed using this reactor produce an effluent which approximates the predicted quality. Moreover, oxygen utilization rates are scaled up with relative accuracy from bench scale reactors to full scale units. There is some difficulty, however, in scaling up and applying coefficients $a$ and $b$ developed from bench scale reactors to a full scale unit because of limitations due to low accuracy of the VSS test, and the difficulty of establishing a solids balance in small scale simulation studies. Using larger reactors of pilot-plant scale enhances the accuracy of these coefficients. Fortunately, the accuracy of coefficients $a$ and $b$ is less important for the designer than those for removal rate constant ($k$) and oxygen demand coefficients ($a'$, $b'$).

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| Reactor no. | Influent average concentration $BOD_5$ $S_o$ (mg/liter) | Effluent average concentration $BOD_5$ $S_e$ (mg/liter) | Average MLVSS concentration $X_{v,a}$ (mg/liter) | Flow rate $Q$ (liter/day) | Oxygen uptake rate $R_r$ [mg $O_2$/(liter)(day)] | Sludge yield $\Delta X_v/V$ [mg sludge/(liter)(day)] | Sludge volume index SVI |
|---|---|---|---|---|---|---|---|
| 1 | 880 | 100 | 3100 | 40.00 | 4025 | 2387 | 75.0 |
| 2 | 870 | 50 | 2800 | 14.90 | 1800 | 806 | 61.0 |
| 3 | 870 | 30 | 3000 | 8.75 | 1292 | 387 | 64.0 |
| 4 | 860 | 20 | 2900 | 3.50 | 780 | 295 | 69.0 |

centration of NVSS is approximately 20 mg/liter. The difference, $30-20=10$ mg/liter, corresponds to NVSS settled in the secondary clarifier. Sludge underflow from the secondary clarifier consists of this NVSS settled *plus* net yield of VSS from reactor operation.

From data in Table 5.4 determine design parameters $k$, $a$, $a'$, $b$, and $b'$. Also estimate nonbiodegradable matter concentration $S_n$ (mg/liter). From Table 5.5 determine $k$ and $S_n$. Also plot column (9) of the table vs. column (3). A graph of this plot is shown in Fig. 5.14.

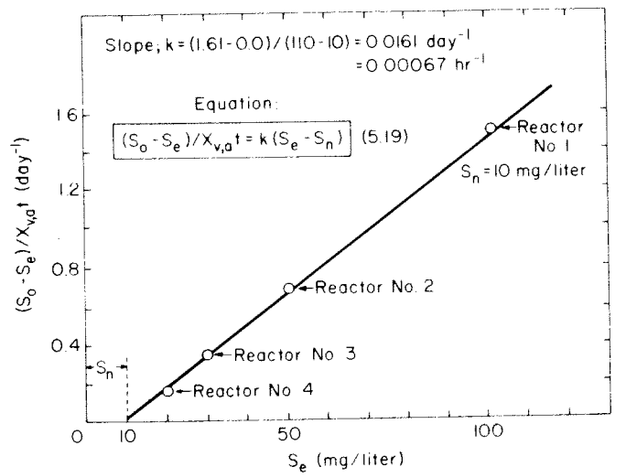



*Fig. 5.14. Graphical determination of k and $S_n$ (Example 5.5).*

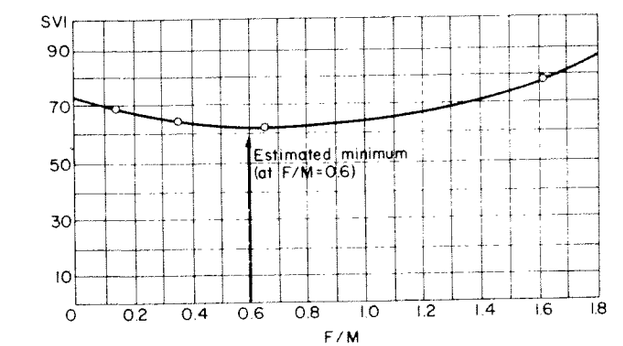



*Fig. 5.15. Plot of SVI vs. F/M (Example 5.5).*

**TABLE 5.5**
**Removal Kinetics[a]**

| | Laboratory data | | | | Calculated data | | | | Lab. data |
|---|---|---|---|---|---|---|---|---|---|
| (1) Reactor no. | (2) $S_o$ (mg/liter) | (3) $S_e$ (mg/liter) | (4) $X_{v,a}$ (mg/liter MLVSS) | (5) $Q$ (liter/day) | (6) Residence time $t = V/Q = 7.0/Q$ (day) | (7) $(7) = (2)-(3)$ $S_o - S_e$ (mg/liter) | (8) $(8) = (4)\times(6)$ $X_{v,a}t$ $\left[\frac{(\text{mg})(\text{day})}{\text{liter}}\right]$ | (9) $(9) = (7)\div(8)$ $(S_o - S_e)/X_{v,a}t$ ($\text{day}^{-1}$) | (10) $(10) = (2)\div(8)$ $F/M = S_o/X_{v,a}t$ $\left[\frac{\text{mg BOD}_5}{(\text{mg MLVSS})(\text{day})}\right]$ | (11) SVI |
| 1 | 880 | 100 | 3100 | 40.00 | 0.175 | 780 | 543 | 1.436 | 1.620 | 75.0 |
| 2 | 870 | 50 | 2800 | 14.90 | 0.470 | 820 | 1316 | 0.623 | 0.661 | 61.0 |
| 3 | 870 | 30 | 3000 | 8.75 | 0.800 | 840 | 2400 | 0.350 | 0.363 | 64.0 |
| 4 | 860 | 20 | 2900 | 3.50 | 2.000 | 840 | 5800 | 0.145 | 0.148 | 69.0 |

[a] Reactor volume is 7 liters.

*Step 1.* Determine the removal rate constant $k$ (kinetics of BOD removal) [Eq. (5.19)].

$$k = 0.0161 \text{ day}^{-1} \quad (0.00067 \text{ hr}^{-1})$$

$$S_n = 10 \text{ mg/liter}$$

A plot of SVI vs. $F/M$ ratio is shown in Fig. 5.15. Estimated minimum SVI occurs at a value of $F/M$ ratio $\approx 0.6$.

*Step 2.* Determine oxygen utilization parameters $a'$ and $b'$ [Eq. (5.86)]. From Table 5.6 determine $a'$ and $b'$.

Plot column (*4*) vs. column (*5*) (Table 5.6). The graph is shown in Fig. 5.16. Then

$$a' = 0.79 \text{ mg } O_2/\text{mg BOD}_r = 0.79 \text{ lb } O_2/\text{lb BOD}_r$$

$$b' = 0.15 \text{ day}^{-1}$$

**TABLE 5.6**
**Oxygen Utilization Parameters**

| | Laboratory data | | Calculated data | |
|---|---|---|---|---|
| (*1*) Reactor no. | (*2*) $X_{v,a}$ (mg/liter) (Table 5.4) | (*3*) $R_r$ [mg $O_2$/(liter)(day)] (Table 5.4) | (*4*) $(4) = (3) \div (2)$ $R_r/X_{v,a}$ ($\text{day}^{-1}$) | (*5*) $(S_o - S_e)/X_{v,a}t$ ($\text{day}^{-1}$) (Table 5.5) |
| 1 | 3100 | 4025 | 1.298 | 1.440 |
| 2 | 2800 | 1800 | 0.643 | 0.620 |
| 3 | 3000 | 1292 | 0.431 | 0.350 |
| 4 | 2900 | 780 | 0.269 | 0.145 |

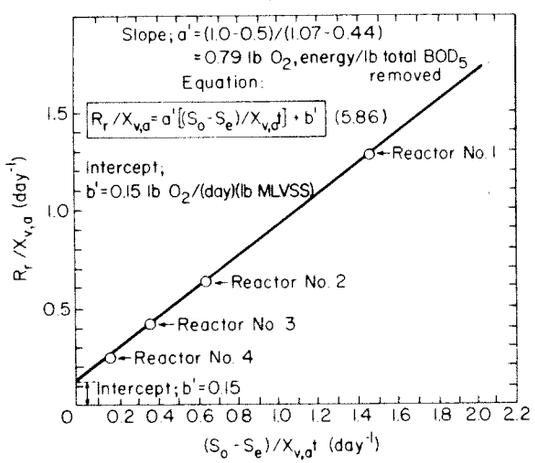



***Fig. 5.16. Graphical determination of a' and b' (Example 5.5).***

**TABLE 5.7**
**Sludge Yield Parameters**

| | Lab. data | Calculated data | | | Lab. data | Calculated data | |
|---|---|---|---|---|---|---|---|
| (*1*) Reactor no. | (*2*) $\Delta X_t/V$ [mg SS/(liter)(day)] (total SS) | (*3*) residence time $t$ (days) (Table 5.5) | (*4*) $(X_{SV,o} - X_{SV,e})/t$ $= 10/t$ [mg SS/(liter)(day)] | (*5*) $(\Delta X_t/V) - (X_{SV,o} - X_{SV,e})/t$ $= (2) - (4)$ [mg MLVSS/(liter)(day)] | (*6*) $X_{v,a}$ (mg/liter) (Table 5.4) | (*7*) $\dfrac{(\Delta X_t/V) - (X_{SV,o} - X_{SV,e})/t}{X_{v,a}}$ [mg MLVSS/(day)(mg MLVSS)] | (*8*) $(S_o - S_e)/X_{v,a}t$ ($\text{day}^{-1}$) (Table 5.5) |
| 1 | 2387 | 0.175 | 57.0 | 2330.0 | 3100 | 0.75 | 1.440 |
| 2 | 806 | 0.470 | 21.3 | 784.7 | 2800 | 0.28 | 0.620 |
| 3 | 387 | 0.800 | 12.5 | 374.5 | 3000 | 0.125 | 0.350 |
| 4 | 295 | 2.000 | 5.0 | 290.0 | 2900 | 0.01 | 0.145 |

*Step 3.* Determine sludge yield parameters $a$ and $b$ [Eq. (5.89)].

$$(X_{NV,o} - X_{NV,e} = 30 - 20 = 10 \text{ mg SS/liter}) \qquad X_{v,o} = 0$$

From Table 5.7 determine $a$ and $b$. Plot column (7) vs. column (8) (Table 5.7). The graph is shown in Fig. 5.17.

$$a = 0.575 \text{ lb MLVSS/lb total BOD}_5 \text{ removed}$$

$$b = 0.075 \text{ lb MLVSS/(day)(lb MLVSS)}$$

SUMMARY Design parameters (Example 5.5)

$$k = 0.0161 \text{ day}^{-1} (0.00067 \text{ hr}^{-1})$$

$$S_n = 10 \text{ mg/liter}$$

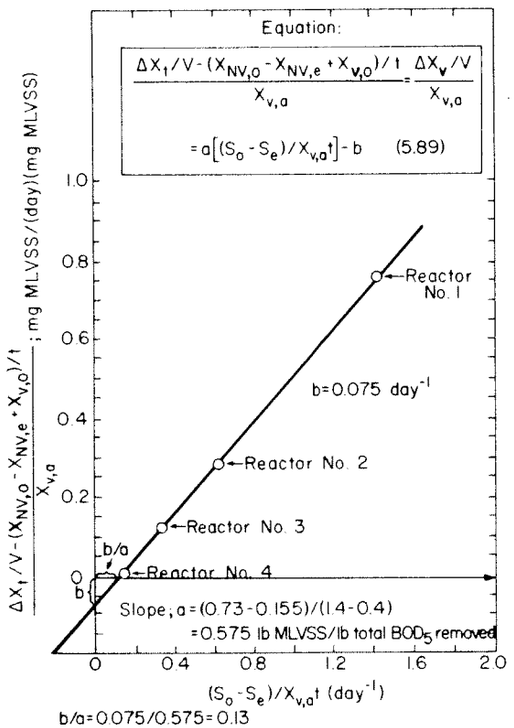



***Fig. 5.17. Graphical determination of a and b (Example 5.5).***



$$a = 0.575 \text{ lb MLVSS/lb total BOD}_5 \text{ removed}$$

$$a' = 0.79 \text{ lb O}_2\text{/lb total BOD}_5 \text{ removed}$$

$$b = 0.075 \text{ lb MLVSS/(day)(lb MLVSS)}$$

$$b' = 0.15 \text{ lb O}_2\text{/(day)(lb MLVSS)}$$

**Example 5.6**

If for a wastewater the lb $O_2$/day required for aerobic biological treatment is

$$\text{lb O}_2\text{/day} = 0.4(\text{lb BOD}_5 \text{ removed/day}) + 0.1(\text{lb MLVSS})$$

write an approximate equation for biological sludge yield in lb/day.

SOLUTION Here

$$a' = 0.4 \qquad (\text{basis BOD}_5, \text{ i.e., } a'_{\text{BOD}_5})$$

$$b' = 0.1$$

The desired equation from Eq. (5.68) is

$$\Delta X_v(\text{lb MLVSS/day}) = a(\text{lb BOD}_5 \text{ removed/day}) - b(\text{lb MLVSS})$$

Approximate relationships for $a$ [Section 4.1.6(c)] and $b$ [from Eq. (5.60)] as functions of $a'$ and $b'$

$$0.82a_{\text{BOD}_5} + 0.58a'_{\text{BOD}_5} = 1.0$$

$$b = b'/1.42$$

Therefore, the approximate value of $a_{\text{BOD}_5}$ is

$$a_{\text{BOD}_5} = (1 - 0.58a'_{\text{BOD}_5})/0.82 = [1 - (0.58)(0.4)]/0.82 = 0.94$$

The approximate value of $b$ is

$$b = b'/1.42 = 0.1/1.42 = 0.07$$

The approximate equation for MLVSS yield is

$$\Delta X_v = 0.94(\text{lb BOD}_5 \text{ removed/day}) - 0.07(\text{lb MLVSS})$$

# 7. Design Procedure for an Activated Sludge Plant

## 7.1. INTRODUCTION

From knowledge of design parameters $k$, $a$, $b$, $a'$, and $b'$, design of the activated sludge plant is undertaken. For the laboratory reactor in Fig. 5.10 there is no recycle of sludge. Net sludge yield is withdrawn intermittently to maintain an average constant concentration ($X_{v,a}$) of MLVSS in the aeration chamber. For the actual plant, sludge is recycled as shown in Fig. 5.1.

A primary variable selected by the designer is concentration $X_{v,a}$ of MLVSS in the aerator. Rate of recycle sludge $Q_R$ is calculated to provide this concentration. Usually $X_{v,a}$ is selected between 2000 and 4000 mg/liter of MLVSS. Another primary variable which is selected by the designer is the concentration $X_{v,u}$ of MLVSS in recycle sludge (stream 7 in Fig. 5.1), which is also equal to MLVSS concentration in underflow from the secondary clarifier [stream 5]. Concentration $X_{v,u}$ is also the same as that in stream 6 (wastage). Good settling sludge is expected to attain a concentration $X_{v,u}$ between 10,000 and 15,000 mg/liter of MLVSS.

At steady state conditions there is no accumulation of sludge. Thus, net yield of sludge in the aerator must be removed in wastage stream 6. For purposes of material balance calculations wastage is assumed to be continuous. In practice, it is usually performed intermittently by the arrangement shown in Fig. 5.18, since it is ordinarily too small to justify continuous withdrawal.

Return and wastage lines are valved as indicated. Valves are actuated by a time clock for intermittent sludge wastage (e.g., 5 min every hour).

## 7.2. MATERIAL BALANCE FOR DETERMINATION OF RECYCLE RATIO OF MLVSS

Write a material balance for MLVSS around the secondary clarifier in Fig. 5.1 (loop · · ·).

| MLVSS, IN | MLVSS, OUT |
|---|---|
| 1. MLVSS in reactor effluent [stream 3] $QX_{v,a}(8.34)$ (lb/day) or [from Eq. (5.5)] $Q_F(1+r)X_{v,a}(8.34)$ (lb/day) | 1. MLVSS in net effluent [stream 4] zero (assuming complete sedimentation of MLVSS in secondary clarifier) |
| | 2. MLVSS in wastage [stream 6] $\Delta X_v + Q_F X_{V,F}$ (8.34) (lb/day) |
| | 3. MLVSS in recycled sludge [stream 7] $Q_R X_{v,u}(8.34) = rQ_F X_{v,u}(8.34)$ (lb/day) |

Then

$$Q_F(1+r)X_{v,a}(8.34) = 0 + \Delta X_v + Q_F X_{V,F}(8.34) + rQ_F X_{v,u}(8.34)$$

Solving for the recycle ratio,

$$r = (8.34Q_F X_{v,a} - \Delta X_v - 8.34Q_F X_{V,F})/[8.34Q_F(X_{v,u} - X_{v,a})] \qquad (5.91)$$

If net sludge yield ($\Delta X_v$) and MLVSS concentration in fresh feed ($X_{V,F}$) are negligible by comparison with term $8.34Q_F X_{v,a}$, Eq. (5.91) simplifies to yield Eq. (5.92).

$$r = X_{v,a}/(X_{v,u} - X_{v,a}) \qquad (5.92)$$



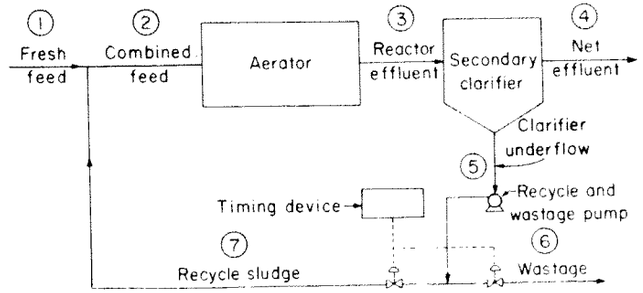



*Fig. 5.18. Arrangement for sludge wastage.*

Wastage flow $Q''$ is calculated by noting that it must contain the $\Delta X_v$ lb/day of net yield of MLVSS *plus* the MLVSS from fresh feed ($Q_F X_{V,F}$). Therefore, since concentration of MLVSS in stream $Q''$ is $X_{v,u}$,

$$\Delta X_v + Q_F X_{V,F}(8.34) = Q'' X_{v,u}(8.34) \qquad (5.93)$$

$$Q'' = (\Delta X_v + 8.34Q_F X_{V,F})/8.34X_{v,u} \qquad (5.94)$$

$Q'$ is obtained by combining Eqs. (5.2) and (5.94):

$$Q' = Q_F - Q'' = Q_F - (\Delta X_v + 8.34QX_{V,F})/8.34X_{v,u} \qquad (5.95)$$

## 7.3. MATERIAL BALANCE FOR CALCULATION OF $S_o$

BOD of combined feed ($S_o$) is calculated by a BOD balance around the junction of fresh feed and recycle sludge to form combined feed, i.e., loop –·–·– in Fig. 5.1.

This material balance is as follows:

BOD IN: $Q_F S_F + Q_R S_e$ or $Q_F S_F + rQ_F S_e$ BOD OUT: $QS_o$ or $Q_F(1+r)S_o$

Then

$$Q_F S_F + rQ_F S_e = Q_F(1+r)S_o$$

Therefore

$$S_o = (S_F + rS_e)/(1+r) \qquad (5.96)$$

From Eq. (5.96) the difference $(S_o - S_e)$ between influent and effluent soluble BOD for the aerator is

$$S_o - S_e = [(S_F + rS_e)/(1+r)] - S_e$$

or

$$S_o - S_e = (S_F - S_e)/(1+r) \qquad (5.97)$$

## 7.4. ALTERNATIVE EXPRESSIONS FOR NET YIELD OF BIOLOGICAL SLUDGE AND OXYGEN UTILIZATION IN THE AERATOR

1. *Net yield of MLVSS.* Substitution of $Q$ and $(S_o - S_e)$ in Eq. (5.68) by their values given by Eqs. (5.5) and (5.97) yields after simplification

$$\Delta X_v = a(S_F - S_e)Q_F - bX_{v,a}V \tag{5.98}$$

Equation (5.98) is an alternative expression for $\Delta X_v$. It is more convenient than Eq. (5.68), since it contains primary variables $S_F$ and $Q_F$ rather than $S_o$ and $Q$. [$S_o$ and $Q$ are calculated from knowledge of $S_F$, $Q_F$, $S_e$, and $r$ from Eqs. (5.96) and (5.5).]

The physical significance of the synthesis term $a(S_F - S_e)Q_F$ is clear. Combined feed $Q$ (Fig. 5.13) is thought of as two hypothetical separate streams (Fig. 5.19). For stream $Q_F$ soluble BOD is reduced from $S_F$ to $S_e$, and biological sludge synthesized as a result of this BOD reduction is $a(S_F - S_e)Q_F$. The other stream ($Q_R$) enters and leaves the reactor with the same unchanged concentration of soluble BOD, i.e., $S_e$. Therefore it *does not* contribute to synthesis of biological sludge.

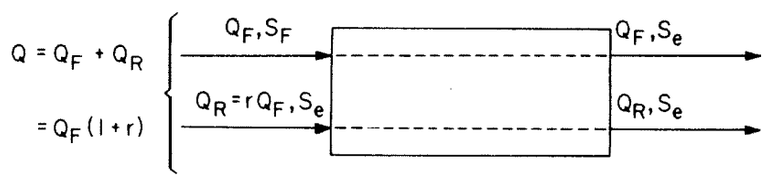



***Fig. 5.19. Diagram corresponding to Eq. (5.98).***

2. *Oxygen utilization in the aerator.* On substitution of $(S_o - S_e)$ and $Q$ by their values given by Eqs. (5.97) and (5.5), respectively, Eq. (5.64) yields

$$\text{lb } O_2/\text{day} = a'(S_F - S_e)Q_F + b'X_{v,a}V \tag{5.99}$$

Significance of energy term $a'(S_F - S_e)Q_F$ is parallel to that of the synthesis term in Eq. (5.98). Only stream $Q_F$ consumes oxygen since stream $Q_R$ enters and leaves the reactor unchanged. Equation (5.99) is more convenient than Eq. (5.64), since it contains primary variables $S_F$ and $Q_F$ rather than $S_o$ and $Q$.

## 7.5. CALCULATION OF RESIDENCE TIME IN REACTOR

Residence time in the reactor is calculated from two criteria in order to determine which one controls the design. These two criteria are

1. Effluent quality, which meets regulatory authority specifications.



Effluent quality depends on substrate removal rate given by Eq. (5.19), which solved for $t$ yields

$$t = (S_o - S_e)/[kX_{v,a}(S_e - S_n)] \tag{5.100}$$

2. Organic loading, evaluated from $F/M$ ratio for optimum flocculation and settling of sludge. This is given by Eq. (5.80), which solved for $t$ yields

$$t = S_o/[X_{v,a}(F/M)] \tag{5.101}$$

Required residence time is calculated from Eqs. (5.100) and (5.101), the larger of the two values of $t$ thus obtained being adopted for design. For wastes which are easily degradable (e.g., sugar refinery, dairy, brewery), the optimum flocculation condition is controlling for residence time calculations. For other wastes, e.g., in petroleum refineries and petrochemical plants, the effluent quality criterion controls residence time requirements since biological degradation is very slow.

## 7.6. EQUATIONS FOR SLUDGE RECYCLE RATIO $r$ IN CASES WHEN EFFLUENT QUALITY AND ORGANIC LOADING CONTROL RESIDENCE TIME

Consider Eq. (5.91) for the sludge recycle ratio. $\Delta X_v$ is given by Eq. (5.68), which is rewritten including the factor 8.34 for use with $Q$ in MGD; $V$ in MG; and $S_o$, $S_e$, and $X_{v,a}$ in mg/liter as

$$\Delta X_v = 8.34a(S_o - S_e)Q - 8.34bX_{v,a}V \tag{5.102}$$

Utilizing Eq. (5.5),

$$\Delta X_v = 8.34a(S_o - S_e)Q_F(1+r) - 8.34bX_{v,a}V \tag{5.103}$$

Since reactor volume $V$ is a value calculated by the designer, it is desirable to rewrite Eq. (5.103) as a function of residence time $t$, which is given by either Eq. (5.100) or (5.101). Term $V$ in Eq. (5.103) is obtained by combining Eqs. (5.17) and (5.5):

$$V = Qt = Q_F(1+r)t \tag{5.104}$$

Substituting in Eq. (5.103) $V$ by its value given by Eq. (5.104),

$$\Delta X_v = 8.34a(S_o - S_e)Q_F(1+r) - 8.34bX_{v,a}Q_F(1+r)t \tag{5.105}$$

Substitution of $\Delta X_v$ given by Eq. (5.105) in Eq. (5.91) yields after simplification

$$r = [X_{v,a} - a(S_o - S_e)(1+r) + bX_{v,a}(1+r)t - X_{v,r}]/(X_{v,u} - X_{v,a}) \tag{5.106}$$

Residence time $t$ on the numerator of Eq. (5.106) is given by either Eq. (5.100) or (5.101), depending on whether residence time is governed by substrate removal rate or optimum flocculation conditions. Equations (5.100) and (5.101) are written in a generalized form as

$$t = (S_o - \alpha)/X_{v,a}\beta \tag{5.107}$$

Where effluent quality controls design (Case 1),

$$\alpha = S_e \tag{5.108}$$

$$\beta = k(S_e - S_n) \tag{5.109}$$

Where optimum flocculation conditions control design (Case 2),

$$\alpha = 0 \tag{5.110}$$

$$\beta = F/M \tag{5.111}$$

Substituting residence time $t$ in the numerator of Eq. (5.106) by its value from Eq. (5.107) yields

$$r = \left[X_{v,a} - a(S_o - S_e)(1+r) + b\frac{S_o - \alpha}{\beta}(1+r) - X_{V,F}\right] \Big/ (X_{v,u} - X_{v,a}) \tag{5.112}$$

Since $S_o$ is *not* a primary variable it is desirable to eliminate it from the numerator of Eq. (5.112). Substituting $(S_o - S_e)$ by its value given in Eq. (5.97), and the value of $S_o$ in term $(S_o - \alpha)$ by its value given in Eq. (5.96),

$$r = \left[X_{v,a} - a(S_F - S_e) + b\frac{S_F + rS_e - \alpha(1+r)}{\beta} - X_{V,F}\right] \Big/ (X_{v,u} - X_{v,a}) \tag{5.113}$$

Now write Eq. (5.113) specifically for Cases (1) and (2).

*For Case (1).* Substituting in Eq. (5.113) $\alpha$ and $\beta$ given by Eqs. (5.108) and (5.109) and simplifying,

$$r = \left[X_{v,a} - a(S_F - S_e) + b\frac{S_F - S_e}{k(S_e - S_n)} - X_{V,F}\right] \Big/ (X_{v,u} - X_{v,a}) \tag{5.114}$$

*Recycle ratio, Case (1):* Effluent quality controls design

*For Case (2).* Substituting in Eq. (5.113) $\alpha$ and $\beta$ given by Eqs. (5.110) and (5.111), solving the resulting expression for $r$, and simplifying [Eq. (5.115)],

$$r = \frac{[X_{v,a} - a(S_F - S_e)](F/M) + bS_F - X_{V,F}(F/M)}{(X_{v,u} - X_{v,a})(F/M) - bS_e} \tag{5.115}$$

*Recycle ratio, Case (2):* Optimum flocculation conditions control design

## 7.7. NEUTRALIZATION REQUIREMENTS

Optimum activity for bacteria occurs at pH values of 6–8. It should be checked if neutralization is needed preceding biological treatment. For alkaline wastes, it is taken as a rule of thumb that up to 0.5 lb of alkalinity (as $CaCO_3$) is removed per lb of BOD removed. This happens because the $CO_2$ evolved from bacterial waste degradation reacts with alkalinity ($OH^-$) present in the waste to form bicarbonate ($HCO_3^-$), which buffers the system at a pH of about 8. Thus, neutralization preceding biological treatment may not be required for some alkaline wastewaters.



## 7.8. NUTRIENT REQUIREMENTS

The appropriate amount of certain nutrients is required for both synthesis and respiration phases of aerobic biological degradation of wastes. Required nutrients include nitrogen, phosphorus, calcium, magnesium, and vitamins. Most of these nutrients, which are required only in trace quantities, are usually present in wastewaters. However, many industrial wastewaters are deficient in nitrogen and phosphorus. Required amounts of nitrogen and phosphorus are estimated by the procedure discussed in this section. If deficiency exists, it is corrected by adding to the wastewater calculated weights of compounds containing nitrogen and phosphorus.

An estimate of requirements for nitrogen and phosphorus is based on the fact that wasted MLVSS ($\Delta X_v$ lb/day) contains approximately 2% of its dry weight as phosphorus and 12% as nitrogen. An estimate of weights of nitrogen and phosphorus to be added comprises

1. Weights of these nutrients which are lost by wastage of MLVSS, i.e.,

Nitrogen: $0.12\ \Delta X_v$ lb/day

Phosphorus: $0.02\ \Delta X_v$ lb/day

2. Weights of these nutrients which are lost in the effluent. (Total effluent $= Q' + Q'' = Q_F$.) Concentrations of soluble nitrogen and phosphorus present in effluent are usually estimated to be 1.0 and 0.5 mg/liter, respectively. Thus, the amounts of nitrogen and phosphorus lost in the effluent are

Nitrogen: $Q_F \times 1.0 \times 8.34$ lb/day

Phosphorus: $Q_F \times 0.5 \times 8.34$ lb/day

where $Q_F$ is the effluent in MGD. Therefore, the total requirements of nitrogen and phosphorus are given by the sum of the estimates made under (1) and (2) [Eqs. (5.116) and (5.117)]:

Nitrogen: $0.12\ \Delta X_v + Q_F \times 1.0 \times 8.34$ lb/day (5.116)

Phosphorus: $0.02\ \Delta X_v + Q_F \times 0.5 \times 8.34$ lb/day (5.117)

In activated sludge plants, nitrogen and phosphorus requirements are provided by the addition of anhydrous or aqueous $NH_3$, $H_3PO_4$, or $(NH_4)_3PO_4$.

## 7.9. DESIGN PROCEDURE FOR ACTIVATED SLUDGE PLANTS

*Step 1.* Calculate the recycle ratio of MLVSS. Select values for $X_{v,a}$ and $X_{v,u}$, usually within the ranges of 2000–4000 and 10,000–15,000 mg/liter, respectively.

From sedimentation and SVI data (Fig. 5.7) select an appropriate value for the $F/M$ ratio. Optimum $F/M$ is usually in the range 0.3–0.7.

The recycle ratio is calculated (1) from Eq. (5.114), which assumes that effluent quality controls the design, and (2) from Eq. (5.115), which assumes that optimum flocculation conditions control the design. The decision on which condition controls design is arrived at in Step 3.

*Step 2.* Calculate BOD of the combined feed ($S_o$). $S_o$ is calculated from Eq. (5.96) utilizing *both* values of $r$ calculated in Step 1(1) and Step 1(2). These two parallel calculations are referred to hence as Steps 2(1) and 2(2), respectively.

*Step 3.* Calculate residence time in the reactor.

*Case (1)* Assuming substrate removal rate controls design [Eq. (5.100)], where $S_o$ is the value calculated in Step 2(1).

*Case (2)* Assuming optimum flocculation conditions control design [Eq. (5.101)], where $S_o$ is the value calculated in Step 2(2).

Possibly* the larger of these two calculated residence times is the one selected for design. Recycle ratio and BOD of combined feed for the specific case which controls design are then adopted. Calculated values for the other case are discarded.

*Step 4.* Calculate the reactor volume. Reactor volume is then calculated from Eq. (5.104) utilizing the value of the residence time selected in Step 3. At this stage, depth of tank is selected. Selection depends on type of aerator utilized (Chapter 4, Sections 14.4, 15.4, and 16.3). Cross-sectional area is then calculated.

*Step 5.* Calculate the net yield of MLVSS. Net yield of MLVSS $\Delta X_v$ is calculated from Eq. (5.105) or Eq. (5.98).

*Note:* A check on the material balance for MLVSS is made at this point. (See Example 5.7 for details.)

*Step 6.* Calculate $Q''$ and $Q'$. $Q''$ and $Q'$ are then calculated from Eqs. (5.94) and (5.95).

*Step 7.* Calculate $\Delta X_{NV}$ and $\Delta X_t$. $\Delta X_{NV}$ is calculated from Eq. (5.72) and total sludge yield is obtained from Eq. (5.1), where $\Delta X_v$ and $\Delta X_{NV}$ are the values calculated in Steps 5 and 7, respectively.

* The reason for the word *possibly* is that frequently a compromise is made in selection of residence time, so that not only reasonable BOD reduction is achieved (i.e., a low value of $S_e$), but also good flocculation conditions for the sludge (although not necessarily the optimum) are obtained.



*Note:* A check on the overall material balance for NVSS is made at this point. (See Example 5.7 for details.)

*Step 8.* Calculate oxygen requirements (lb/day) from Eq. (5.64) or Eq. (5.99).

*Step 9.* Specify aeration equipment. Aeration equipment is selected from oxygen requirements determined in Step 8 and manufacturer's specifications for aerators. This procedure is described in Chapter 4, Sections 14, 15, and 16. Calculated values needed are (1) total HP and number of aeration units; (2) power level, HP/1000 gal; and (3) spacing between aerators. A layout for aerators in the tank is selected. (Details are given in Example 4.5.)

*Step 10.* Check neutralization requirements. Verify if neutralization is required prior to biological treatment. For alkaline wastes, the rule of thumb stating that up to 0.5 lb of alkalinity (as $CaCO_3$) is removed per lb of BOD removed is frequently employed (Section 7.7).

*Step 11.* Evaluate nutrient requirements. Requirements for nitrogen and phosphorus (lb/day) are evaluated as described in Section 7.8 from Eqs. (5.116) and (5.117).

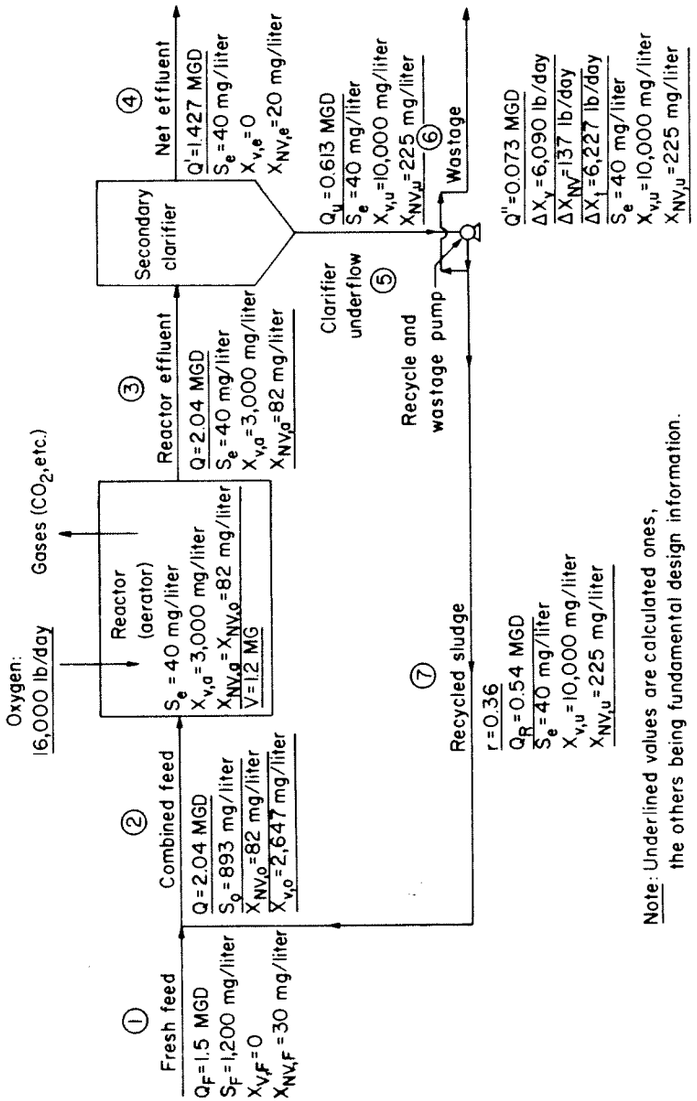
## 7.10. NUMERICAL EXAMPLE: DESIGN OF AN ACTIVATED SLUDGE PLANT

**Example 5.7**

An industrial plant (Example 5.5) considers an activated sludge system for treatment of their wastewaters. Base design on the following data in addition to information given in Example 5.5:

Flow: 1.5 MGD (1,500,000 gal/day)
Influent $BOD_5$: 1200 mg/liter
Effluent $BOD_5$: 40 mg/liter
$OH^-$ alkalinity of raw wastewater: 90 mg/liter (as $CaCO_3$)
Total Kjeldahl nitrogen and phosphorus in fresh feed: 85 and 3 mg/liter, respectively
$C_{SW}$: Saturation DO of wastewater at the temperature and barometric pressure of the test
$C_{SW}$ (summer conditions at 30°C): 7.4 mg/liter
$C_{SW}$ (winter conditions at 18°C): 10.3 mg/liter
$\alpha = K_La$(wastewater)/[$K_La$(water)] = 0.72
Assume operating DO (level in aeration basin $\approx$ 1.0 mg/liter)
Characteristics of surface aerators: given by Fig. 4.17
Select for design purposes:
$X_{v,a}$ = 3000 mg/liter of MLVSS
$X_{v,u}$ = 10,000 mg/liter of MLVSS
Neglect VSS concentration in effluent from secondary clarifier and in fresh feed

SOLUTION A flowsheet for the proposed activated sludge plant is shown in Fig. 5.20. Follow design procedure summarized in Section 7.9.




Note: Underlined values are calculated ones, the others being fundamental design information.

*Fig. 5.20. Flow diagram for Example 5.7.*



*Step 1.* Calculate the recycle ratio of MLVSS.

(1) Assume substrate removal rate controls design [Eq. (5.114)]. Here

$X_{v,a}$ = 3000 mg/liter

$a(S_F - S_e)$ = 0.575(1200 − 40) = 0.575 × 1160 = 665 mg/liter

$X_{v,u} - X_{v,a}$ = 10,000 − 3000 = 7000 mg/liter

$X_{v,a} - a(S_F - S_e)$ = 3000 − 665 = 2335 mg/liter

$X_{V,F}$ = 0

Thus Eq. (5.114) yields

$$r = [2335 + b(S_F - S_e)/k(S_e - S_n)]/7000$$

Calculate separately term $b(S_F - S_e)/k(S_e - S_n)$, which is the only different term in equations for $r$ in Step 1(1) and Step 1(2).

$$b(S_F - S_e)/k(S_e - S_n) = 0.075(1200 - 40)/0.0161(40 - 10) = 180$$

$$\therefore r = (2335 + 180)/7000 = 0.36$$

(2) Assume optimum flocculation conditions control design [Eq. (5.115)]. Here

$[X_{v,a} - a(S_F - S_e)]$ = 2335 [calculated Step 1(1)]

$(X_{v,u} - X_{v,a})$ = 7000 [also calculated Step 1(1)]

$X_{V,F}(F/M)$ = 0 since $X_{V,F}$ = 0

and

$$(F/M) = 0.6$$

$$b = 0.075$$

$$S_e = 40$$

Thus Eq. (5.115) yields

$$r = (2335 \times 0.6 + 0.075 \times 1200)/(7000 \times 0.6 - 0.075 \times 40) = 0.353$$

*Step 2.* Calculate $S_o$ from Eq. (5.96).

(1) Assume substrate removal rate controls design (i.e., $r$ = 0.36).

$$S_o = (1200 + 0.36 \times 40)/(1 + 0.36) = 893 \text{ mg/liter}$$

(2) Assume optimum flocculation conditions control design (i.e., $r$ = 0.353).

$$S_o = (1200 + 0.353 \times 40)/(1 + 0.353) = 900 \text{ mg/liter}$$

*Step 3.* Calculate residence time.

(1) Assume substrate removal rate controls design ($S_o$ = 893 mg/liter). From Eq. (5.100)

$$t = (893-40)/[0.0161 \times 3000(40-10)] = 0.59 \text{ day}$$

or

$$24 \times 0.59 = 14.2 \text{ hr}$$

(2) Assume optimum flocculation conditions control design ($S_o$ = 900 mg/liter).
From Eq. (5.101)

$$t = 900/3000 \times 0.6 = 0.5 \text{ day}$$

or

$$24 \times 0.5 = 12 \text{ hr}$$

Thus substrate removal rate controls design, and calculations in part (2) for Steps 1–3 are discarded. $F/M$ ratio for the reactor is [from Eq. (5.80)]

$$F/M = 893/3000 \times 0.59 = 0.504$$

From Fig. 5.15, this value of $F/M$ is compatible with good flocculation conditions for the sludge. Therefore, no adjustment of selected residence time (14.2 hr) is required to achieve compatibility between BOD reduction and good flocculation conditions.

*Step 4.* Calculate reactor volume. Throughput rate [Eq. (5.5)]:

$$Q = 1.5(1+0.36) = 2.04 \text{ MGD}$$

Reactor volume [Eq. (5.104)]:

$$V = 2.04 \times 0.59 = 1.2 \text{ MG}$$

or

$$1{,}200{,}000 \text{ gal} \times \text{ft}^3/7.48 \text{ gal} = 161{,}000 \text{ ft}^3$$

For depths of 10 and 15 ft, for example, corresponding cross-sectional areas are

$$\text{For } d = 10 \text{ ft} \qquad A = 161{,}000/10 = 16{,}100 \text{ ft}^2$$

or

$$16{,}100 \text{ ft}^2 \times \text{acre}/43{,}500 \text{ ft}^2 = 0.37 \text{ acre}$$

$$\text{For } d = 15 \text{ ft} \qquad A = 161{,}000/15 = 10{,}700 \text{ ft}^2$$

or

$$10{,}700/43{,}500 = 0.246 \text{ acre}$$

Parallel basins might be recommended.



*Step 5.* Calculate net yield of MLVSS. From Eq. (5.105) ($a$, $b$ from Example 5.5),

$$\Delta X_v = 0.575(893-40) \times 2.04 \times 8.34 - 0.075 \times 3000 \times 1.2 \times 8.34$$

$$\Delta X_v = 8342 - 2252 = 6090 \text{ lb/day}$$

Or from Eq. (5.98),

$$\Delta X_v = 0.575(1200-40) \times 1.5 \times 8.34 - 0.075 \times 3000 \times 1.2 \times 8.34$$

$$\Delta X_v = 8342 - 2252 = 6090 \text{ lb/day}$$

*Note:* At this point a check on material balance calculations is made. Calculate concentration of VSS in combined feed to the reactor ($X_{v,o}$) from Eq. (5.75), where $X_{V,F} = 0$.

$$X_{v,o} = (0 + 0.36 \times 10{,}000)/(1+0.36) = 2647 \text{ mg/liter}$$

The difference between concentrations of MLVSS in reactor effluent (3000 mg/liter) and the value 2647 mg/liter in reactor influent must correspond to the net yield of MLVSS (i.e., $\Delta X_v$ = 6090 lb/day). Therefore, 3000 − 2647 = 353 mg/liter, i.e., 353 mg of MLVSS are produced per liter of liquor flowing through the reactor. Then based on flow $Q$ = 2.04 MGD, net production of MLVSS is $353 \times 2.04 \times 8.34 \approx 6006$ lb/day, which agrees approximately with the value 6090 lb/day of $\Delta X_v$ calculated in Step 5 (within 1.4%).

*Step 6.* Calculate $Q''$ and $Q'$ [Eqs. (5.94) and (5.95), respectively].

$$Q'' = 6090/8.34 \times 10{,}000 = 0.073 \text{ MGD} \qquad \text{for } X_{V,F} = 0$$

or

$$Q'' = 73{,}000 \text{ gal/day} \qquad (9.9 \text{ gal/min})$$

and

$$Q' = 1{,}500{,}000 - 73{,}000 = 1{,}427{,}000 \text{ gal/day}$$

*Step 7.* Calculate $\Delta X_{NV}$ and $\Delta X_t$. From Eq. (5.72),

$$\Delta X_{NV} = 1.5(30-20) \times 8.34 + 0.073 \times 20 \times 8.34 = 125 + 12.2 \approx 137 \text{ lb/day}$$

The total sludge yield $\Delta X_t$ is [from Eq. (5.1), where $X_{V,F} = 0$]

$$\Delta X_t = 6090 + 137 = 6227 \text{ lb/day}$$

*Note:* Check on material balance for NVSS in the influent.

$$\text{IN} = Q_F X_{NV,F} = 1.5 \times 30 \times 8.34 = 376 \text{ lb/day}$$

$$\text{OUT} = Q'' X_{NV,u} = \Delta X_{NV} = 137 \text{ lb/day}$$

$$Q' X_{NV,e} = 1.427 \times 20 \times 8.34 = 239 \text{ lb/day (checks)}$$

$$X_{NV,u} = \Delta X_{NV}/8.34 Q'' = 137/8.34 \times 0.073 = 225 \text{ mg/liter}$$

From Eq. (5.74),

$$X_{NV,o} = (30 + 0.36 \times 225)/(1+0.36) = 82 \text{ mg/liter}$$

*Step 8.* Calculate oxygen requirements from either Eq. (5.64) or Eq. (5.99). From Eq. (5.64) ($a'$, $b'$ from Example 5.5)

$$\text{lb } O_2/\text{day} = 0.79(893-40) \times 2.04 \times 8.34 + 0.15 \times 3000 \times 1.2 \times 8.34$$
$$= 11{,}500 + 4500 = 16{,}000$$

From Eq. (5.99)

$$\text{lb } O_2/\text{day} = 0.79(1200-40) \times 1.5 \times 8.34 + 0.15 \times 3000 \times 1.2 \times 8.34$$
$$= 11{,}500 + 4500 = 16{,}000$$

or

$$16{,}000/24 = 665 \text{ lb } O_2/\text{hr}$$

*Step 9.* Specify aeration equipment. Specification of aeration equipment and layout for aerators in this application are presented in Chapter 4 (Example 4.5).

*Step 10.* Check neutralization requirements. Utilize rule of thumb: 0.5 lb of alkalinity (as $CaCO_3$) are removed per lb of BOD removed. Calculate lb of BOD removed per day.

$$(1200-40) \times 1.5 \times 8.34 = 14{,}512 \text{ lb/day}$$

Thus 14,512/2 = 7256 lb/day of alkalinity are removed. Calculate the lb/day of alkalinity in fresh feed.

$$90 \times 1.5 \times 8.34 = 1126 \text{ lb/day}$$

Since $1126 < 7256$, no neutralization is required prior to the biological process.

*Step 11.* Evaluate nutrient requirements.

Nitrogen

1. Nitrogen lost from system through wastage of sludge:

$$0.12\,\Delta X_v = 0.12 \times 6090 = 730 \text{ lb/day}$$

2. Nitrogen lost in effluent (1.0 mg/liter):

$$1 \times 1.5 \times 8.34 \quad \text{(total nitrogen required)} \approx \frac{13 \text{ lb/day}}{743 \text{ lb/day}}$$

Nitrogen available is (85 mg/liter)

$$85 \times 1.5 \times 8.34 = 1070 \text{ lb/day}$$

Thus addition of nitrogen is *not* required.

Phosphorus

1. Phosphorus lost from system through wastage of sludge:

$$0.02\,\Delta X_v = 0.02 \times 6090 = 121.8 \text{ lb/day}$$



2. Phosphorus lost in the effluent ($\approx 0.5$ mg/liter):

$$0.5 \times 1.5 \times 8.34 \approx \frac{6 \text{ lb/day}}{128 \text{ lb/day}}$$

Phosphorus available is (3 mg/liter)

$$3 \times 1.5 \times 8.34 = 37.6 \text{ lb/day}$$

Thus $128 - 37.6 \approx 91.0$ lb/day of phosphorus should be added.

# 8. The Michaelis–Menten Relationship

## 8.1. DERIVATION OF MICHAELIS–MENTEN RELATIONSHIP

Formulation of the Michaelis–Menten relationship is based on studies of pure cultures. However, it is used in determining kinetics of substrate degradation by a heterogeneous population of microorganisms, which is the case for the activated sludge process.

Degradation of wastes by microorganisms is accomplished through a complex series of chemical reactions. These reactions are catalyzed by organic catalysts (enzymes) present in the microorganisms. Enzymes are large molecular weight compounds. Usually enzymes are quite specific in their functions as catalysts, i.e., a given enzyme ordinarily catalyzes a specific chemical reaction. Bacteria contains a great variety of enzymes, each one being responsible for a minor step in the complex process of biological metabolism.

The action of enzymes is represented by the following chemical equation:

$$\text{Substrate} + \text{Enzyme} \rightleftharpoons \text{Enzyme-Substrate complex} \rightleftharpoons \text{Enzyme} + \text{Products}$$

or symbolically

$$[S] + [E] \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} [E\text{–}S] \underset{k_{-2}}{\overset{k_2}{\rightleftharpoons}} [E] + [P] \tag{5.118}$$

where $k$'s stand for the reaction rate constants. As indicated by Eq. (5.118), substrate and enzyme unite to form an enzyme–substrate complex. This is followed by the breaking down of this complex, resulting in the end products. The enzyme remains unchanged and is ready to reenter the reaction, acting therefore as a catalyst.

The rate of substrate removal is obtained from Eq. (5.118) by making the assumption that the breaking down of the enzyme–substrate complex is irreversible. Then Eq. (5.118) is rewritten as

$$[S] + [E] \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} [E\text{–}S] \xrightarrow{k_2} [E] + [P] \tag{5.119}$$

This assumption is essentially correct if measurements are taken shortly after introduction of substrate, which means that very little product has been allowed to form. Under these circumstances, since the rate of the inverse reaction

$$[E] + [P] \xrightarrow{k_{-2}} [E\text{–}S]$$

is given by*

$$k_{-2}[E][P]$$

and since $[P] \approx 0$, it may be assumed that the breaking down of the enzyme–substrate complex is irreversible. Therefore Eq. (5.118) is rewritten as shown [Eq. (5.119)]. The rate of reaction measured under these conditions is that occurring immediately on contact of substrate and microorganism, and is referred to as the initial rate of reaction. To develop kinetic data it is necessary to measure a series of such initial rates, corresponding to different concentrations of substrate shortly after the substrate samples are brought into contact with the microorganism.

Substrate removal rate is denoted here by $V$. For a batch reactor, it corresponds to the slope of the BOD curve in Fig. 5.3 (Section 3.1) at any specified time $t$, corresponding to a concentration $S$ of substrate. A specific substrate removal rate per mg/liter of MLVSS is utilized, i.e. [Eq. (5.120)],

$$V = -(1/X_{v,a})(dS/dt) \tag{5.120}$$

(Minus sign is necessary since $dS/dt < 0$ and $V > 0$.)

For the continuous reactor as shown in Section 3.2, this corresponds to [in finite rather than differential form]

$$(S_o - S_e)/X_{v,a}t$$

where $t$ is the residence time in the continuous reactor. The substrate removal rate is equal to the rate of formation of product P, and is given by Eq. (5.121).

$$V = k_2[E\text{–}S] \tag{5.121}$$

Similarly, the rate of formation of the enzyme–substrate complex (E–S) is

$$\text{Rate of formation of (E–S)} = k_1[S][E] \tag{5.122}$$

The rate of conversion of enzyme–substrate complex to E and S is [Eq. (5.123)]

$$\text{Rate of conversion of (E–S)} = k_{-1}[E\text{–}S] \tag{5.123}$$

Therefore, the net change of concentration of enzyme–substrate complex is

$$d[E\text{–}S]/dt = \underbrace{k_1[S][E]}_{\text{formation}} - \underbrace{k_{-1}[E\text{–}S]}_{\text{destruction}} - \underbrace{k_2[E\text{–}S]}_{\text{destruction}} \tag{5.124}$$

* In the formulation to follow, symbols [S], [E], [E–S], and [P] are used to denote concentrations of substrate, enzyme, enzyme–substrate complex, and products, respectively.



Let the total concentration of enzyme in the reacting system be denoted as $E_t$. This includes not only free enzyme (E) but also enzyme in combined form as enzyme–substrate complex (E–S), i.e. [Eqs. (5.125) and (5.126)],

$$[E_t] = [E] + [E\text{–}S] \tag{5.125}$$

$$\therefore [E] = [E_t] - [E\text{–}S] \tag{5.126}$$

Substituting [E] in Eq. (5.124) by its value given in Eq. (5.126) yields

$$d[E\text{–}S]/dt = k_1([E_t] - [E\text{–}S])[S] - k_{-1}[E\text{–}S] - k_2[E\text{–}S] \tag{5.127}$$

At steady state conditions it is usually assumed that concentration of intermediate complexes (enzyme–substrate complex in this case) remains unchanged. This assumption is called the steady state approximation. Therefore

$$d[E\text{–}S]/dt = 0 \tag{5.128}$$

and Eq. (5.127) becomes Eq. (5.129).

$$k_1([E_t] - [E\text{–}S])[S] - k_{-1}[E\text{–}S] - k_2[E\text{–}S] = 0 \tag{5.129}$$

Solving for [E–S],

$$[E\text{–}S] = \frac{[E_t][S]}{[S] + (k_{-1} + k_2)/k_1} \tag{5.130}$$

Term $(k_{-1} + k_2)/k_1$ is called the Michaelis–Menten constant and is designated as $K_s$.

$$K_s = (k_{-1} + k_2)/k_1 \tag{5.131}$$

Then, Eq. (5.130) is rewritten as Eq. (5.132).

$$[E\text{–}S] = \frac{[E_t][S]}{[S] + K_s} \tag{5.132}$$

Substituting this value in Eq. (5.121), the following expression for the substrate removal rate $V$ is obtained:

$$V = k_2\frac{[E_t][S]}{[S] + K_s} \quad \text{(Michaelis–Menten relationship)} \tag{5.133}$$

## 8.2. COROLLARIES OF MICHAELIS–MENTEN RELATIONSHIP

The two corollaries stated in Section 3.1 are obtained from Eq. (5.133).

**Corollary 1:** High substrate concentrations

At high substrate concentrations,

$$[S] \gg K_s \tag{5.134}$$

Neglecting $K_s$ in the denominator of Eq. (5.133) as compared to [S] and simplifying,

$$V = k_2[E_t] = V_{MAX} \tag{5.135}$$

Equation (5.135) indicates that at high substrate concentrations, removal of substrate takes place at a maximum rate ($V_{MAX}$) independent of concentration. It is assumed that at these high substrate concentrations all active sites of the enzymes are saturated with substrate, and so reaction proceeds as fast as possible independent of substrate concentration (zero-order reaction). This corresponds to the section of the BOD curve in Fig. 5.3 (Section 3.1) from time zero up to time $t'$, where the tangent to the BOD curve essentially coincides with the curve itself (constant slope).

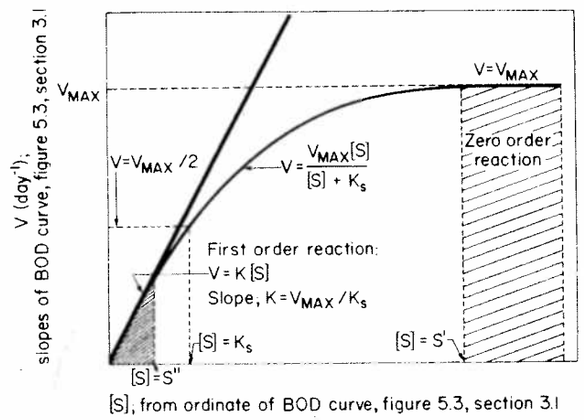



***Fig. 5.21. Plot of V vs. [S].***

From Fig. 5.3, a plot of slopes of the BOD curve ($V$'s or substrate removal rates) vs. the corresponding BOD values ([S]) can be constructed (Fig. 5.21). The region of high substrate concentration encompasses values from the right-hand extremity of the graph down to a concentration $S'$ (corresponding to time $t'$ in Fig. 5.3). This is the region where $V = V_{MAX}$ = constant irrespective of substrate concentration.

From Eq. (5.135) Eq. (5.133) is rewritten as

$$V = V_{MAX}\frac{[S]}{[S]+K_s} \tag{5.136}$$

**Corollary 2:** Low substrate concentrations

At low substrate concentrations,

$$[S] \ll K_s \tag{5.137}$$



Neglecting [S] in the denominator of Eq. (5.136) as compared to $K_s$,

$$V = V_{MAX}[S]/K_s \tag{5.138}$$

Since $V_{MAX}$ and $K_s$ are both constant for a specific waste, Eq. (5.138) is rewritten as

$$V = K[S] \tag{5.139}$$

where

$$K = V_{MAX}/K_s \tag{5.140}$$

Equation (5.139) indicates that at low substrate concentrations, substrate removal follows first-order kinetics. In Fig. 5.21 this corresponds to the section of the curve from a value of the abscissa $S = 0$ up to a value $S''$. In this section, the curve is essentially replaced by a straight line passing through the origin (with slope = $K$). This situation corresponds to that encountered in continuous biological reactors operating at steady state conditions. In fact, Fig. 5.21 up to [S] = $S''$ is identical to Fig. 5.5 (Section 3.2), which was utilized for determination of the removal rate constant from a series of continuous laboratory reactors operating in parallel. Had these experiments been continued on higher concentrations of substrate, the straight line would have become a curve like the one in Fig. 5.21. However, operation of continuous reactors is always conducted at substrate concentrations much below 500 mg/liter (expressed as $BOD_5$). Under these conditions the straight line relationship applies.

## 8.3. SIGNIFICANCE OF MICHAELIS–MENTEN CONSTANT $K_s$

From Eq. (5.136) it is shown that $K_s$ is equal to the substrate concentration when substrate removal rate $V$ equals *half* the maximum, i.e., when $V = V_{MAX}/2$. This is shown by letting $V = V_{MAX}/2$ in Eq. (5.136) and solving for [S]. The final result is

$$[S] = K_s \qquad (\text{for } V = V_{MAX}/2)$$

This is indicated in Fig. 5.21.

## 8.4. DETERMINATION OF $V_{MAX}$: THE LINEWEAVER–BURK PLOT

The value of $V_{MAX}$ estimated from Fig. 5.21 is inaccurate since it is an asymptotic value. A better way of determining $V_{MAX}$ is as follows. Take the inverse of Eq. (5.136),

$$1/V = (K_s/V_{MAX})(1/[S]) + (1/V_{MAX}) \tag{5.141}$$

Based on Eq. (5.141) a linear plot of $1/V$ vs. $1/[S]$ is constructed, from which

the characteristic constants $V_{MAX}$ and $K_s$ are determined from the slope and intercept of the straight line. This graph, which is shown in Fig. 5.22, is known as the Lineweaver–Burk plot [5]. As indicated the intercept at the abscissa corresponds to $(-1/K_s)$, since for $1/V = 0$ one obtains $1/[S] = -1/K_s$ from Eq. (5.141).

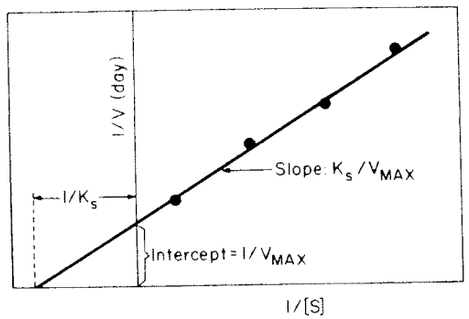



*Fig. 5.22. Lineweaver–Burk plot.*

## 8.5. MICHAELIS–MENTEN RELATIONSHIP WHEN NONBIODEGRADABLE MATTER IS PRESENT IN THE SYSTEM

If the concentration of nonbiodegradable matter is indicated as $[S_n]$, it is accounted for by substituting the value of [S] by $([S]-[S_n])$ in Eq. (5.136). A similar substitution in Section 3.2 led to Eq. (5.19) from Eq. (5.18).

Therefore, modified Eq. (5.136) is

$$V = V_{MAX}\frac{[S]-[S_n]}{K_s+[S]-[S_n]} \tag{5.142}$$

The two corollaries studied in Section 8.2 derived from Eq. (5.133) are also obtained from Eq. (5.142). Similarly, Fig. 5.21 is replotted when nonbiodegradable matter is present (Fig. 5.23).

From Eq. (5.142), it is shown that

$$K_s = [S] - [S_n]$$

when $V = V_{MAX}/2$

The Lineweaver–Burk equation when nonbiodegradable matter is present is written by replacing [S] in Eq. (5.141) by $([S]-[S_n])$. The corresponding Lineweaver–Burk plot follows directly from the equation thus obtained.



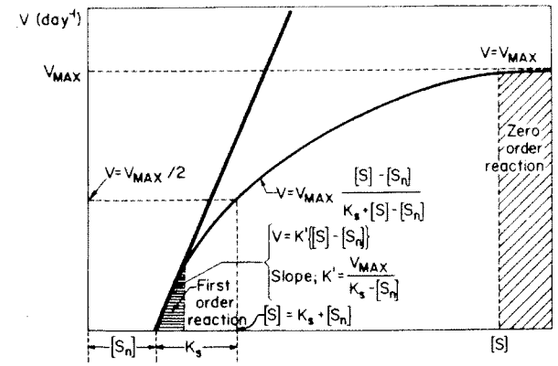



*Fig. 5.23. Plot of V vs. [S] when nonbiodegradable matter is present.*

## 8.6. MICHAELIS–MENTEN RELATIONSHIP IN TERMS OF SPECIFIC GROWTH RATE OF SLUDGE

The Michaelis–Menten relationship [Eq. (5.136)] is written as a function of the specific substrate removal rate [Eq. (5.120)]. An equivalent form is written as a function of the specific growth rate of sludge, defined as

$$\mu = (1/X_{v,a})(dX/dt) \tag{5.143}$$

Whereas values of $dS/dt$ in Eq. (5.120) correspond to slopes of the BOD curve in Fig. 5.3, values of $dX/dt$ in Eq. (5.143) correspond to slopes of the MLVSS curve.

Assuming that the specific growth rate of sludge is proportional to the specific substrate removal rate, i.e., that a constant fraction of the substrate removed is converted to cells ($\mu = aV$), Eq. (5.136) is rewritten as

$$\mu = \mu_{MAX}\frac{[S]}{[S]+K_s} \tag{5.144}$$

From Eq. (5.144) it is shown, following the same procedure utilized in Section 8.3 for Eq. (5.136), that $K_s$ is equal to the substrate concentration when the specific growth rate of the sludge is equal to half the maximum specific growth rate, i.e., $K_s = [S]$, when $\mu = \mu_{MAX}/2$. All corollaries, derivations, and graphical constructions studied in Sections 8.2 to 8.5 based on the specific substrate removal rate are also applicable in terms of the specific growth rate of the sludge.

# 9. The Concept of Sludge Age

Sludge age is defined as the mean residence time of MLVSS in the reactor. For the activated sludge plant shown in Fig. 5.1 this corresponds to

$$t_S = \text{lb MLVSS in reactor/net output of VSS from the system (lb/day)} \tag{5.145}$$

or

$$t_S = \frac{X_{v,a}V}{\text{total lb/day of VSS wasted, i.e., output of VSS (lb/day)} - \text{input of VSS in fresh feed (lb/day)}} \tag{5.146}$$

or

$$t_S = \frac{X_{v,a}V}{(\Delta X_v + Q'X_{v,e}) - Q_F X_{V,F}} \quad \text{(days)} \tag{5.147}$$

In Eq. (5.147) numerator ($X_{v,a}V$) equals total lb of MLVSS in the reactor at any time (a constant value at steady state conditions). Terms between parentheses in the denominator represent total VSS wasted, including sludge wasted purposely ($\Delta X_v$) and that lost in effluent from the secondary clarifier ($Q'X_{v,e}$). Term $Q_F X_{V,F}$ corresponds to input of sludge in fresh feed. The difference between the terms within parentheses and this value $Q_F X_{V,F}$ represents net output of VSS from this system.

Since in the formulation of the activated sludge process concentration of VSS in effluent from the secondary clarifier is neglected (i.e., $X_{v,e} \approx 0$), Eq. (5.147) reduces to Eq. (5.148).

$$t_S = X_{v,a}V/(\Delta X_v - Q_F X_{V,F}) \qquad (X_{v,e} = 0) \tag{5.148}$$

Finally, when concentration of VSS in fresh feed is also negligible (i.e., $X_{V,F} \approx 0$),

$$t_S = X_{v,a}V/\Delta X_v \qquad (X_{v,e} \approx 0,\ X_{V,F} \approx 0) \tag{5.149}$$

Sludge age is also referred to as mean cell residence time or solids retention time. The relationship between sludge age and hydraulic or liquid retention time ($t = V/Q$) is presented for two types of complete mix reactors: (1) complete mix—no recycle reactor; and (2) complete mix reactor with recycle (a) with wastage directly from reactor (or reactor effluent), and (b) with wastage from the sludge recycle line.

1. *Complete mix—no recycle reactor.* In this model, liquor in the reactor unit is completely mixed and there is no recycle. This *does not* correspond to the conventional activated sludge process, but rather to flow-through devices such as aerated lagoons (Chapter 6, Section 5), assuming complete mixing to occur in the lagoon. The situation is depicted by Fig. 5.24.



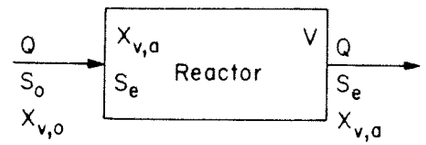



*Fig. 5.24. Complete mix reactor without recycle.*

Hydraulic or liquid retention time is $t = V/Q$, and the sludge age is [from Eq. (5.145)]

$$t_S = X_{v,a}V/Q(X_{v,a} - X_{v,o}) = [X_{v,a}/(X_{v,a} - X_{v,o})]t \tag{5.150}$$

Frequently, concentration of VSS in influent is negligible (i.e., $X_{v,o} \approx 0$). Then, Eq. (5.150) reduces to

$$t_S = X_{v,a}V/QX_{v,a} = V/Q = t \qquad (X_{v,o} \approx 0) \tag{5.151}$$

Thus for the complete mix reactor without recycle when concentration of VSS in influent is negligible, sludge age equals hydraulic (or liquid retention) time. Concentration of sludge in the reactor is kept at a constant value $X_{v,a}$. Since concentration of sludge in effluent also equals $X_{v,a}$, it follows that residence time is such that sludge is not washed out from the system faster than it can reproduce. In fact, since steady state is assumed (constant $X_{v,a}$ in reactor and effluent), residence time is such that sludge washed out in effluent is *exactly replaced* by an equal mass of net sludge yield for the same time interval.

2. *Complete mix reactor with recycle.* This model corresponds to the conventional activated sludge process (Fig. 5.1). Wastage of sludge is usually accomplished (Fig. 5.1) by drawing off from the sludge recycle line. However, the possibility of wasting sludge directly from the reactor (or reactor effluent) is also considered.

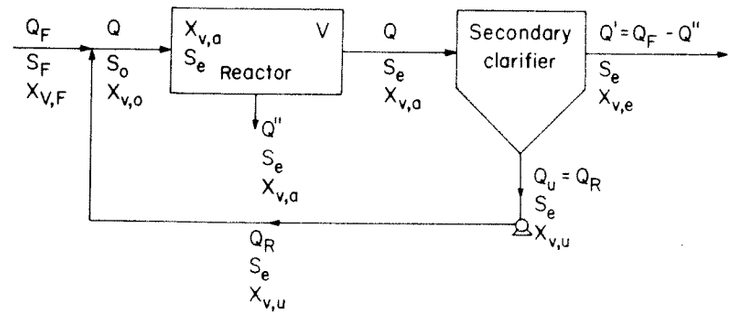



*Fig. 5.25. Diagram of complete mix reactor with recycle, and wastage directly from the reactor.*

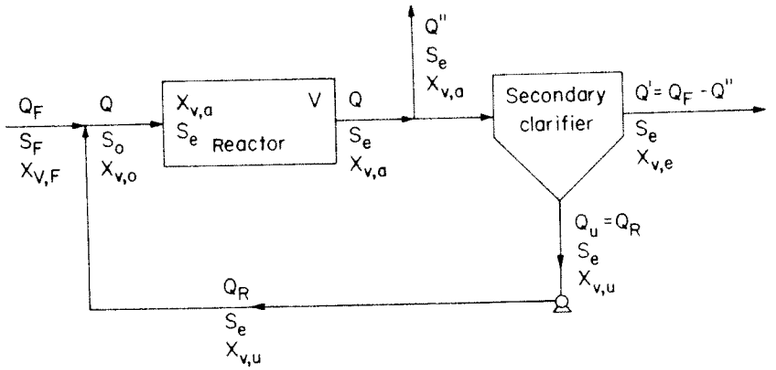



***Fig. 5.26. Diagram of complete mix reactor with recycle, and wastage directly from the reactor effluent.***

a. Complete mix reactor with recycle, and wastage directly from reactor (or reactor effluent). This corresponds to diagrams shown in Figs. 5.25 and 5.26, the former with wastage taken directly from reactor and the latter with wastage taken from reactor effluent. The hydraulic or liquid retention time for the models in these figures is $t = V/Q$, whereas sludge is [from Eq. (5.145)]

$$t_S = \frac{X_{v,a}V}{\text{lb/day VSS wasted in } Q'' + \text{lb/day VSS lost in effluent from secondary clarifier} - \text{input of VSS in fresh feed (lb/day)}} \tag{5.152}$$

or

$$t_S = \frac{X_{v,a}V}{[Q''X_{v,a} + (Q_F - Q'')X_{v,e}] - Q_F X_{V,F}} \tag{5.153}$$

If the concentration of VSS in the effluent from the secondary clarifier is negligible (i.e., $X_{v,e} \approx 0$), Eq. (5.153) yields

$$t_S = X_{v,a}V/(Q''X_{v,a} - Q_F X_{V,F}) \qquad (X_{v,e} \approx 0) \tag{5.154}$$

Finally, when the concentration of VSS in fresh feed is also negligible (i.e., $X_{V,F} \approx 0$),

$$t_S = X_{v,a}V/Q''X_{v,a} = V/Q'' \qquad (X_{v,e} \approx 0;\ X_{V,F} \approx 0) \tag{5.155}$$

Comparing Eqs. (5.17) and (5.155), it follows that since $Q'' \ll Q$, then

$$t_S \gg t \tag{5.156}$$

b. Complete mix reactor with recycle, and wastage from recycle line. This corresponds to the flow diagram in Fig. 5.1. Since concentration of sludge in the wastage stream is equal to $X_{v,u}$, whereas it is $X_{v,a}(X_{v,a} < X_{v,u})$ when



wastage is taken directly from the reactor or reactor effluent (Figs. 5.25 and 5.26), it follows that the volumetric wastage flow $Q''$ (which contains a total of $\Delta X_{v,a}$ lb/hr of sludge) is *less* for the case of Fig. 5.1. This is one advantage of taking wastage directly from the recycle line. Hydraulic or liquid retention time is $t = V/Q$, whereas sludge age is given from Eq. (5.152) as

$$t_S = \frac{X_{v,a}V}{[Q''X_{v,u} + (Q_F - Q'')X_{v,e}] - Q_F X_{V,F}} \tag{5.157}$$

If the concentration of VSS in the effluent from the secondary clarifier is negligible (i.e., $X_{v,e} \approx 0$), Eq. (5.157) yields

$$t_S = X_{v,a}V/(Q''X_{v,u} - Q_F X_{V,F}) \qquad (X_{v,e} \approx 0) \tag{5.158}$$

Finally, when concentration of VSS in fresh feed is also negligible (i.e., $X_{V,F} \approx 0$),

$$t_S = X_{v,a}V/Q''X_{v,u} \qquad (X_{v,e} \approx 0;\ X_{V,F} \approx 0) \tag{5.159}$$

Consequently, when wastage is taken from the recycle line, knowledge of both mixed liquor and recycled sludge microorganism concentrations are required for calculation of sludge age.

For the complete mix reactor with recycle (Figs. 5.25, 5.26, and 5.1), residence time is such that sludge is not wasted from the system faster than it reproduces. In fact, since a steady state condition is assumed, wastage ($\Delta X_v$) equals exactly the net sludge yield for the same time interval if loss of VSS in the effluent from the secondary clarifier is negligible.

**Example 5.8**

For the activated sludge plant designed in Example 5.7 calculate the sludge age.

SOLUTION This is a case of a complete mix reactor with recycle, wastage being taken from the recycle line. Concentration of VSS in the secondary clarifier effluent is negligible (i.e., $X_{v,e} \approx 0$), and also $X_{V,F} = 0$. Equation (5.159) is then utilized to calculate the sludge age. Here

$X_{v,a}$ = 3000 mg/liter

$X_{v,u}$ = 10,000 mg/liter

$V$ = 1.2 MG

$Q''$ = 0.073 MGD

Then from Eq. (5.159)

$$t_S = 3000 \times 1.2/0.073 \times 10{,}000 = 4.43 \text{ days}$$

Hydraulic residence time $t$ is 14.2 hr (Example 5.7, Section 7.10).

A relationship between sludge age, substrate removal rate $[(S_o - S_e)/X_{v,a}t]$, and parameters $a$ and $b$ for sludge yield is written from Eq. (5.149) for the complete mix reactor with recycle. [In Eq. (5.149) it is assumed that concentrations of VSS in the effluent from the secondary clarifier and in fresh feed are negligible.]

If in Eq. (5.149) net sludge yield $\Delta X_v$ is replaced for the value given by Eq. (5.68),

$$t_S = X_{v,a}V/[a(S_o - S_e)Q - bX_{v,a}V]$$

and

$$1/t_S = [a(S_o - S_e)Q - bX_{v,a}V]/X_{v,a}V$$

Since $V/Q = t$, then

$$1/t_S = a[(S_o - S_e)/X_{v,a}t] - b \qquad (5.160)$$

# 10. Kinetics of Continuous Treatment Systems: Plug Flow, Complete Mix, and Arbitrary Flow Reactors

In the formation of the activated sludge process, the model utilized for the continuous reactor was that of a complete mix vessel. The plug flow continuous reactor model was only briefly mentioned in Section 5 (Fig. 5.9). In this section three models for the continuous reactor (Fig. 5.27) are described: (1) plug flow reactor, (2) complete mix reactor, and (3) arbitrary flow reactor.

1. *Plug flow reactor.* In the plug flow reactor fluid particles travel through the vessel without mixing and therefore are discharged in the same sequence in which they enter. If a continuous tracer is introduced starting at time $t = 0$ (concentration of tracer in the influent being $C_o$), no tracer appears in effluent until a time $t_r$, equal to theoretical residence time of the fluid in the vessel, has elapsed. Then, the concentration of tracer in the effluent jumps from a zero value to the value $C_0$ and remains at that value as long as continuous injection of tracer is maintained.

If a first dose of slug tracer is introduced at time $t = 0$, none of it appears in the effluent until a time $t_r$ has elapsed. At $t = t_r$, concentration of tracer in the effluent jumps from zero to $C_0$. At time $(t_r + dt)$ it is back again to zero. It jumps again to $C_0$ at time $t_r + \Delta t$, where $\Delta t$ is the time interval between the first two discontinuous injections of tracer.

2. *Complete mix reactor.* In this reactor immediate dispersion of particles takes place as they enter the vessel. For a continuous tracer, its concentration



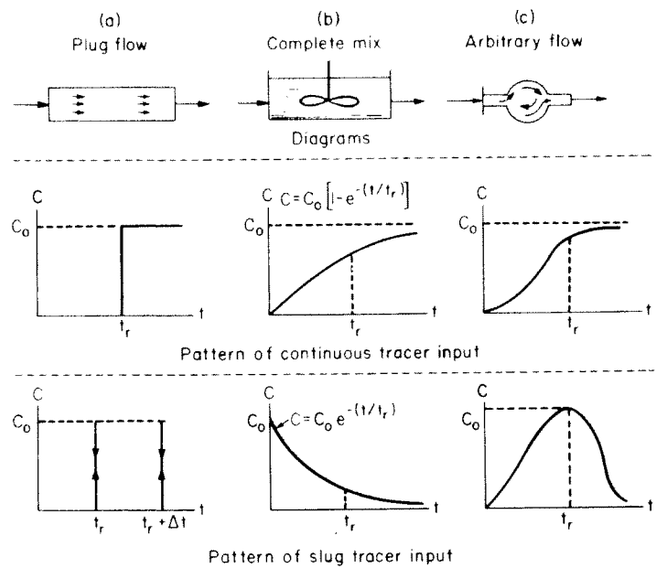



*Fig. 5.27. Continuous reactor models (adapted from Ref. [7]).*

in the effluent as a function of time is determined by the following material balance for tracer around the reactor:

Rate of change in amount of tracer in reactor
= rate of input of tracer to reactor − rate of output of tracer from reactor

or

$$(dC/dt)V = QC_0 - QC \qquad (5.161)$$

where $C$ is the effluent concentration of tracer at any time $t$; $V$ the volume of the reactor; $Q$ the flow rate; and $C_0$ the concentration of tracer in the influent. From Eq. (5.161),

$$dC/dt = (Q/V)(C_0 - C) \qquad (5.162)$$

Since $V/Q = t_r$ (hydraulic residence time, which is denoted here as $t_r$ so as to distinguish it for time variable $t$), Eq. (5.162) yields

$$dC/dt = (1/t_r)(C_0 - C)$$

or

$$dC/(C_0 - C) = (1/t_r)\,dt \qquad (5.163)$$

Integrating Eq. (5.163) and solving for $C$,

$$C = C_0[1 - e^{-(t/t_r)}] \qquad (5.164)$$

This corresponds to the curve for the concentration of continuous tracer shown in Fig. 5.27(b). As steady state conditions are approached (theoretically at $t = \infty$), Eq. (5.164) yields

$$C = C_0$$

Thus, the curve approaches asymptotically the ordinate $C = C_0$. If addition of tracer stops when a steady state condition is reached, the corresponding value for tracer concentration in the effluent drops gradually following curve $C = C_0 e^{(-t/t_r)}$, also shown in Fig. 5.27(b). This corresponds to a reactor being purged of tracer. As $t \to \infty$ (steady state), the concentration of tracer in effluent approaches zero.

3. *Arbitrary flow reactor.* These reactors correspond to a partial mix condition between plug flow and complete mix types. Typical patterns for continuous and slug tracer input for arbitrary flow reactors are shown in Fig. 5.27(c). Mathematical analysis of this type of reactor is considerably more complicated than plug or complete mix types, and for this reason these two models are usually chosen to describe reactor performance.

It is interesting to compare efficiency of BOD removal for continuous reactors with recycle (typical activated sludge plant), adopting complete mix and plug flow models to describe the reactor in question. Comparison is made by computing for a given wastewater (i.e., $k$ and $S_n$ fixed) the effluent BOD ($S_e$) for fixed values of flow rate $Q$, influent BOD ($S_F$), recycle ratio ($r$), and MLVSS concentration ($X_{v,a}$) for various assumed residence times $t$. For the complete mix reactor, the kinetic model is given by Eq. (5.19). If in Eq. (5.19) $S_o$ is eliminated utilizing Eq. (5.96), the result is

$$(S_F - S_e)/[(1+r)X_{v,a}t] = k(S_e - S_n) \tag{5.165}$$

Solving for $S_e$,

$$S_e = [S_F + kS_n(1+r)X_{v,a}t]/[1 + kX_{v,a}(1+r)t] \tag{5.166}$$

A typical plot of $S_e$ vs. $t$ obtained from Eq. (5.166) is shown by the curve labeled "complete mix model" indicated in Fig. 5.28. (For $t = 0$, $S_e = S_F$ and for $t = \infty$, $S_e = S_n$.)

A kinetic model for the continuous reactor with recycle under plug flow conditions is mathematically quite difficult to derive. A model has been obtained, however, by Lawrence and McCarthy [4]. This model predicts for a given residence time $t$ a lower value of effluent BOD than that for the corresponding complete mix model. This is indicated by the dotted curve in Fig. 5.28. Thus, the plug flow recycle system is theoretically more efficient than the complete mix recycle system for stabilization of soluble wastes. In practice, however, the plug flow model is difficult to obtain because of longitudinal dispersion. Also, the complete mix systems handle sudden changes in influent



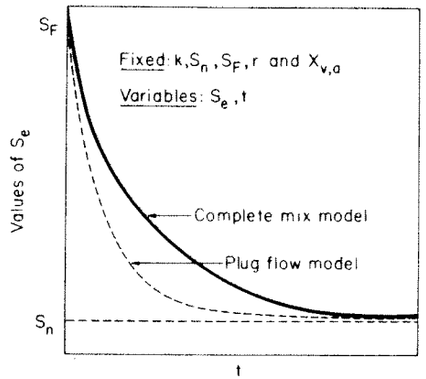



***Fig. 5.28. Plot of $S_e$ vs. t for continuous flow reactors with recycle (complete mix and plug flow models).***

BOD (shock loads) much more satisfactorily than plug flow systems. In addition, there is the unfavorable situation of variable $F/M$ ratios along plug flow reactors, and its possible undesirable effect on the settling characteristics of the sludge discussed in Section 5. All these factors tend to reduce differences in actual efficiency of BOD removal for the two models.

Figure 5.29 shows the progressive BOD reduction occurring in a plug flow reactor from value $S_o$ at the inlet to the final value $S_e$. By dividing the aeration tank into a series of complete mix reactors (assume a uniform soluble BOD value for the liquor between any two dotted partitions in Fig. 5.29), an improvement in treatment performance is obtained without a major loss in ability of the system to handle shock loads. This is the idea behind the step aeration scheme (Chapter 6, Section 4.1, Fig. 6.6).

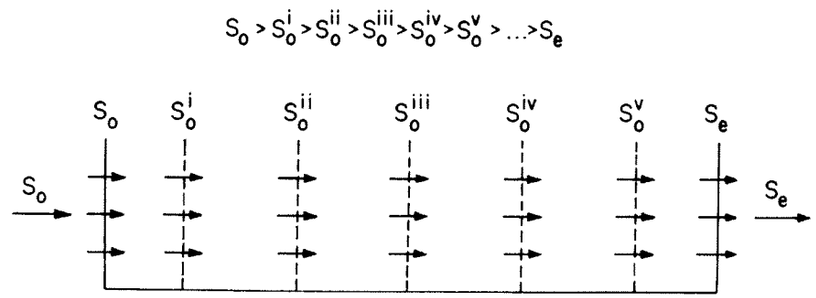



***Fig. 5.29. BOD reduction in a plug flow reactor.***

## Problems

**I.** Determination of design parameters for an activated sludge project. An industrial plant considers an activated sludge system for disposal of wastewaters. Preliminary tests are performed in laboratory scale continuous reactors. Four reactors are operated in parallel until steady state conditions are obtained. Data taken are presented in the following tabulations.

**TABLE 1**
**For Removal Kinetics**

| Reactor no. | Average $BOD_5$ of influent (mg/liter) | Average $BOD_5$ of effluent (mg/liter) | Average MLVSS concentration (mg/liter) | Residence time (hr) |
|---|---|---|---|---|
| 1 | 850 | 100 | 2000 | 4.81 |
| 2 | 800 | 50 | 2500 | 7.32 |
| 3 | 750 | 25 | 3100 | 12.7 |
| 4 | 700 | 15 | 3100 | 18.4 |

**TABLE 2**
**Oxygen Utilization and Sludge Production**

| Reactor no. | Oxygen uptake rate $R_r$ [mg $O_2$/(liter)(day)] | Sludge yield $\Delta X_v/V$ [mg sludge/(liter)(day)] |
|---|---|---|
| 1 | 3200 | 2500 |
| 2 | 2187 | 1450 |
| 3 | 1425 | 780 |
| 4 | 1008 | 403 |

From these data determine design parameters $k$ ($hr^{-1}$ and $day^{-1}$), $S_n$, $a$, $a'$, $b$, and $b'$.

**II.** An organic chemical wastewater is to be treated by a proposed activated sludge plant to produce an effluent BOD of 50 mg/liter during summer conditions (20°C). Wastewater characteristics are

Flow = 2.0 MGD
Influent BOD = 1000 mg/liter

Treatment parameters are

$k = 0.0005\ hr^{-1}$ at 20°C
$a = 0.50$ lb MLVSS/lb $BOD_r$
$a' = 0.55$ lb $O_2$/lb $BOD_r$
$b = 0.1$ lb MLVSS/(day)(lb MLVSS)
$b' = 0.14$ lb $O_2$/(day)(lb MLVSS)
$F/M = 0.6$
$S_n = 0.0$ mg/liter



Take

$X_a = 3000$ mg/liter
$X_u = 12{,}000$ mg/liter

Neglect influent suspended solids.

Calculate

1. Reactor volume (Mgal) and sludge return rate (Mgal/day)
2. Oxygen required (lb $O_2$/hr)
3. Net sludge yield (lb MLVSS/day)
4. HP required for surface aeration. Characteristics of the aerator are given by Fig. 4.17.

Base calculation on 20°C operation and take

$C_{sw} = 8.0$ mg/liter
$C_L = 1.0$ mg/liter
$\alpha = 0.8$

Calculate required power level in HP/1000 gal

5. Nutrient requirements (lb/day) for nitrogen and phosphorus

## References


1. Eckenfelder, W. W., Jr., "Industrial Pollution Control." McGraw-Hill, New York, 1966.
2. Eckenfelder, W. W., Jr., "Water Quality Engineering for Practicing Engineers." Barnes & Noble, New York, 1970.
3. Eckenfelder, W. W., Jr., and Ford, D. L., "Water Pollution Control." Pemberton Press, Austin and New York, 1970.
4. Lawrence, A. W., and McCarthy, P. L., *J. Sanit. Eng. Div., Am. Soc. Civ. Eng.* **49**, SA3 (1970).
5. Lineweaver, H., and Burk, D., *J. Am. Chem. Soc.* **56**, 658–666 (1934).
6. "Manual for Biooxidation Unit." Biodevelopment Associates, P.O. Box 1752, Austin, Texas.
7. Metcalf & Eddy, Inc., "Wastewater Engineering: Collection, Treatment, Disposal." McGraw-Hill, New York, 1972.